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Date: March 13, 2025

To: Chairman Beltz, Members of the House Agriculture Committee.

NDSPLS is made up of over 100 members, many of which are Registered Professional Land Surveyors who live and practice in the State of North Dakota. There are approximately 500 Registered Land Surveyors who are licensed to practice in ND. We are licensed and regulated by the North Dakota State Board of Registration for Professional Engineers and Professional Land Surveyors. Our mission and objective is: to unite all of the Professional Land Surveyors in the State of North Dakota; to elevate the standards of the surveying profession; to establish basic minimum standards and requirements for surveys; to assist in promoting legislative and educational programs to improve the professional status of the Land Surveyor; to work in cooperation with local, county, state, federal and tribal governments in our field of endeavor; to uphold a rigid code of ethics; to strive to improve our relations with our clients and the public by doing our work with precision and integrity; and to maintain a good relationship between Land Surveyors and Engineers.

The North Dakota Society of Professional Land Surveyors is IN FAVOR of Senate Bill No. 2260.

The purpose of this bill is to codify state plane coordinate (SPC) systems corresponding to the new North American Terrestrial Reference Frame of 2022 (NATRF2022). SPCs are used by the surveying, mapping, and engineering professions to map features within the state using a common reference frame. SPCs corresponding to the North American Datum of 1927 (NAD27) and the North American Datum of 1983 (NAD83) are presently codified under this section. The advances in GPS technology and computing have enabled scientists and geodesists to gain a better understanding of the shape of the earth. In light of these advancements, the National Geodetic Survey (NGS) has developed the NATRF2022 for implementation throughout the nation, and similar initiatives to enact updated SPC systems are being pursued.

To gain a better understanding of the reasons for creating and adopting NATRF2022, knowing the history of the North American Datums is helpful. The initial origins of the datums in the US were primarily used to map the coastal shorelines for the purposes of navigation. As the United States developed and the need for large-scale infrastructure projects increased – i.e. railroads, highways, dams, etc. - having accurate large-scale maps to plan these projects became increasingly important. Triangulation lines were run on corridors throughout the country to measure and extend precise geographic control (see diagram 1). Using these triangulation lines, NAD27 was developed utilizing the Clarke Spheroid of 1866 (Clarke 1866) as the mathematical shape of the Earth, which was based primarily on observations in North America. Also, due to the lack of computing power, the statistical calculations used in the reduction of the NAD27 positions were broken up throughout the country. While Clarke 1866 was highly advanced for its time, as technology developed and global considerations were considered in mathematical models, the deficiencies in NAD27 became increasingly known. Also, as the country developed, the control network needed to be extended into areas falling outside of the original triangulation corridors. As additional triangulation lines were run, the geodesists continued to gain a better understanding of the shape of the earth. Lastly, other countries throughout the world conducted similar control networks, all of which enriched the general understanding of the shape of the earth.

In 1974, the NGS initiated a new datum project. The new datum project was to use a global model for the shape of the earth known as an "ellipsoid" and was to use one, statistical reduction encompassing the North American Continent. This datum became known as NAD83 and was based on the GRS80 ellipsoid. While the new NAD83 data utilized computing technology for its reduction, the bulk of the data was based on traditional, terrestrial methods and was developed without the benefit of GPS technology. Once again, the development of GPS technology created a more refined understanding of the size and shape of the earth. As the GPS technology developed, NGS provided adjustments to the NAD83 datum to account for these developments in understanding. Also during this time, the Department of Defense developed the World Geodetic System of 1984 (WGS84), which is almost identical to NAD83. (See diagram 2 showing the differences between NAD27, NAD83, and WGS84.)

Now having the benefit of many years of continuously collected GPS data and the ability to reduce this information with robust computing power, NGS has developed the NATRF2022 model. The NATRF2022 model is highly advanced and takes into account the latest understandings of the shape of the earth. Additionally, due to the continuously collected GPS data, NATRF2022 also accounts for a greater understanding of continental shift.

NAD coordinates are typically expressed as geographic coordinates – i.e. latitude, longitude, and height. For the purposes of surveying, mapping, and engineering, the use of projected cartesian grid coordinates is preferable – i.e. northing, easting, and elevation. The grid calculations are much simpler than geographic calculations and are better suited for surveying and engineering applications. For this reason, there are SPC systems codified in our law. These SPC positions correspond with the geographic coordinates per both the NAD27 and NAD83 datums. In short, these SPC systems convert the geographic positions on an ellipsoidal surface to a plane surface. As to be expected, when forcing a round surface to a flat plane, there is distortion. The larger the surface to be flattened, the more distortion that occurs. To minimize this distortion, the state is divided into two zones, north and south, for both the NAD27 and NAD83 datums. (See diagram 3 for a map of the existing zones.)

Since the NATRF2022 datum will soon be available, there is a need to define new SPCs that correspond with the new datum. In 2018, NGS solicited public comment in the beginning stages of the development of the SPC zones. In 2019, the NGS committee selected the NDDOT to assist in developing the general concept of zones in North Dakota. Also in 2019, the NDDOT hired a consultant to develop the proposed zones. In 2020, the consultant completed the conceptual designs for the zones and were considered by stakeholders. The stakeholders included the NDDOT, NDSPLS, the Department of Water Resources, and the North Dakota GIS Technical Committee. After some refinements based on public input, the zones were formally submitted to NGS for review and approval.

The SPC systems envisioned by this new law include one, statewide zone and 16 low distortion zones. (See diagram 4 for the statewide zone, see diagram 5 for the 16 low distortion zones.) The purpose of the statewide zone is to create a statewide reference frame so there is one zone for large-scale mapping throughout the state. The concept of the statewide zone was based on stakeholder input that, for GIS applications where map distortion is not as much of a concern, a statewide zone would be beneficial. In fact, it was learned that the existing NAD83 south zone was fulfilling that purpose already. The purpose of the 16 low distortion zones is more suited for surveying and engineering applications where actual ground distance measurements are important. As shown on diagrams 4 and 5, the distortion estimates for the statewide zone and the 16 low distortion zones are colorized. Overall, NDSPLS believes the design of the statewide zone and the 16 low distortion zones is very well thought-out and considers the needs of all the stakeholders.

Turning to the development of the proposed legislation under consideration, the consensus of NDSPLS was that the language of the existing codes has served the state and the profession well since the NAD27 datum was originally codified in 1975. In 1989, the same fundamental language was used to add the parameters of the NAD83 datum while keeping the parameters of the NAD27 datum in effect. Now in 2025, after considering different alternatives, the consensus of NDSPLS is to codify the NATRF2022 zones in a similar manner to the codification of the previous zones. Additionally, NDSPLS believes that keeping the prior zones in code is important as a tremendous amount of surveying has been done over the years using both the NAD27 and NAD83 datums. Ultimately, the purpose of this bill is to add the new NATRF2022 zones for their use and application going forward, and at the same time, maintain the parameters of the historically used datums for legacy purposes.

Therefore, NDSPLS urges a <u>DO PASS</u> on Senate Bill No. 2260.

Respectfully submitted,

Aaron Hummert, PLS
Chair of the Legislative Committee
North Dakota Society of Professional Land Surveyors

Reference: Elithorp, James A., Jr. & Findorff, Dennis D. (2003) Geodesy For Geomatics and GIS Professionals, Ann Arbor, MI, Copley Custom Textbooks: An imprint of XanEdu Custom Publishing.

Diagram 1: Map showing the NAD27 triangulation lines.

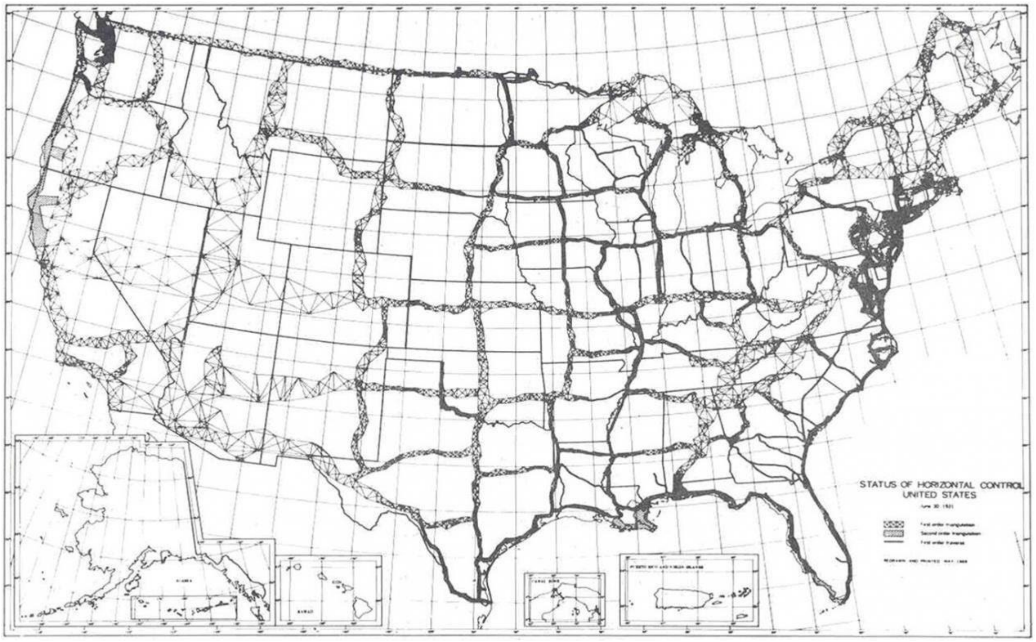


Diagram 2: Map showing the differences between NAD27, NAD83, and WGS84.

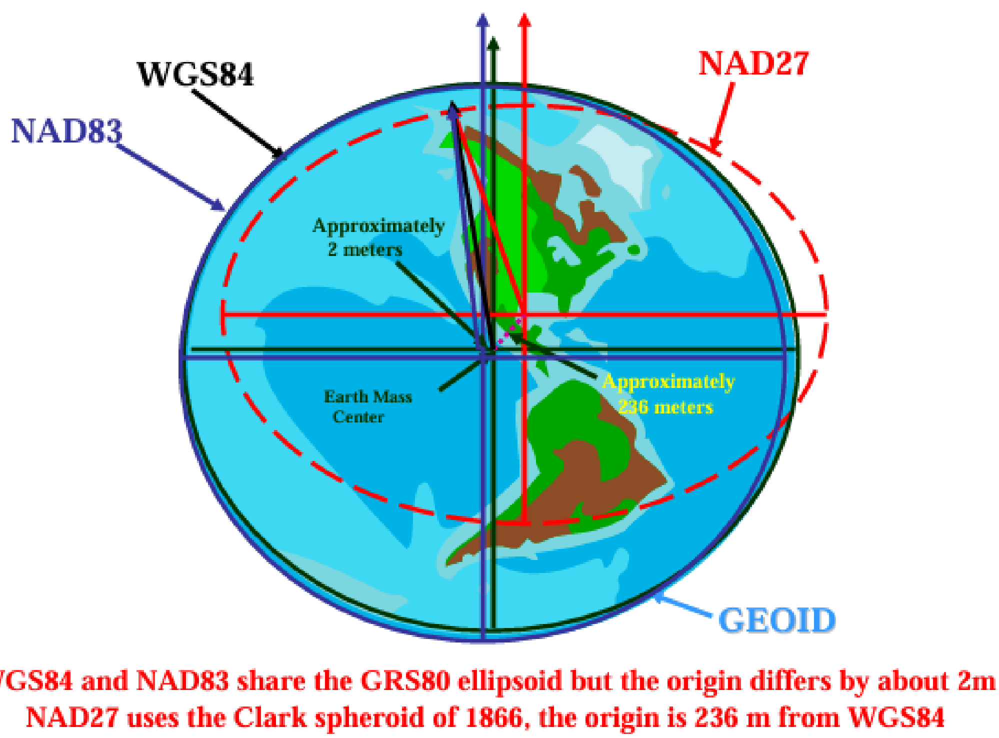


Diagram 3: Map of the existing NAD27 and NAD83 north and south zones.

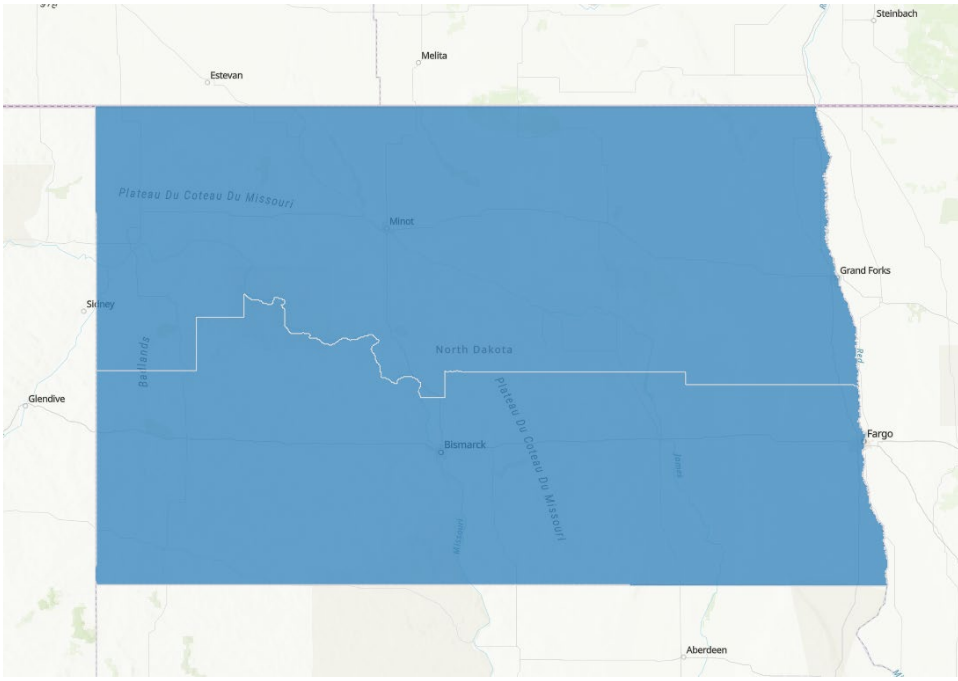


Diagram 4: Map of the statewide zone.

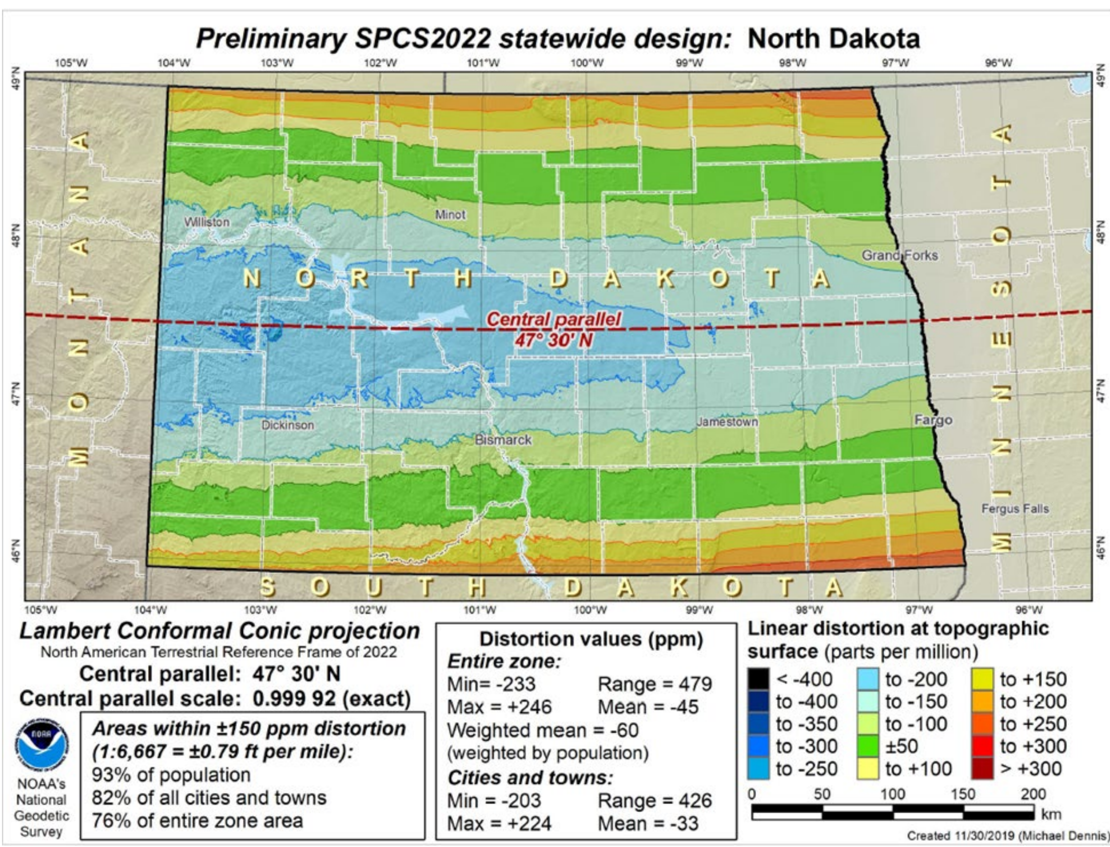


Diagram 5: Map of the 16 low distortion zones.

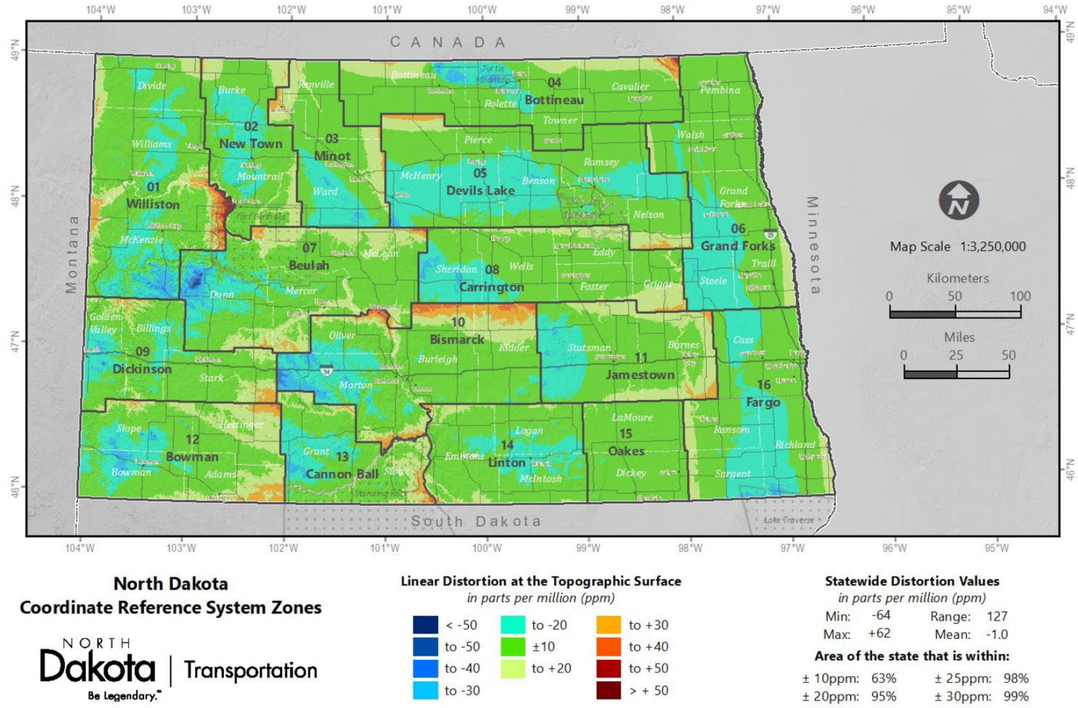
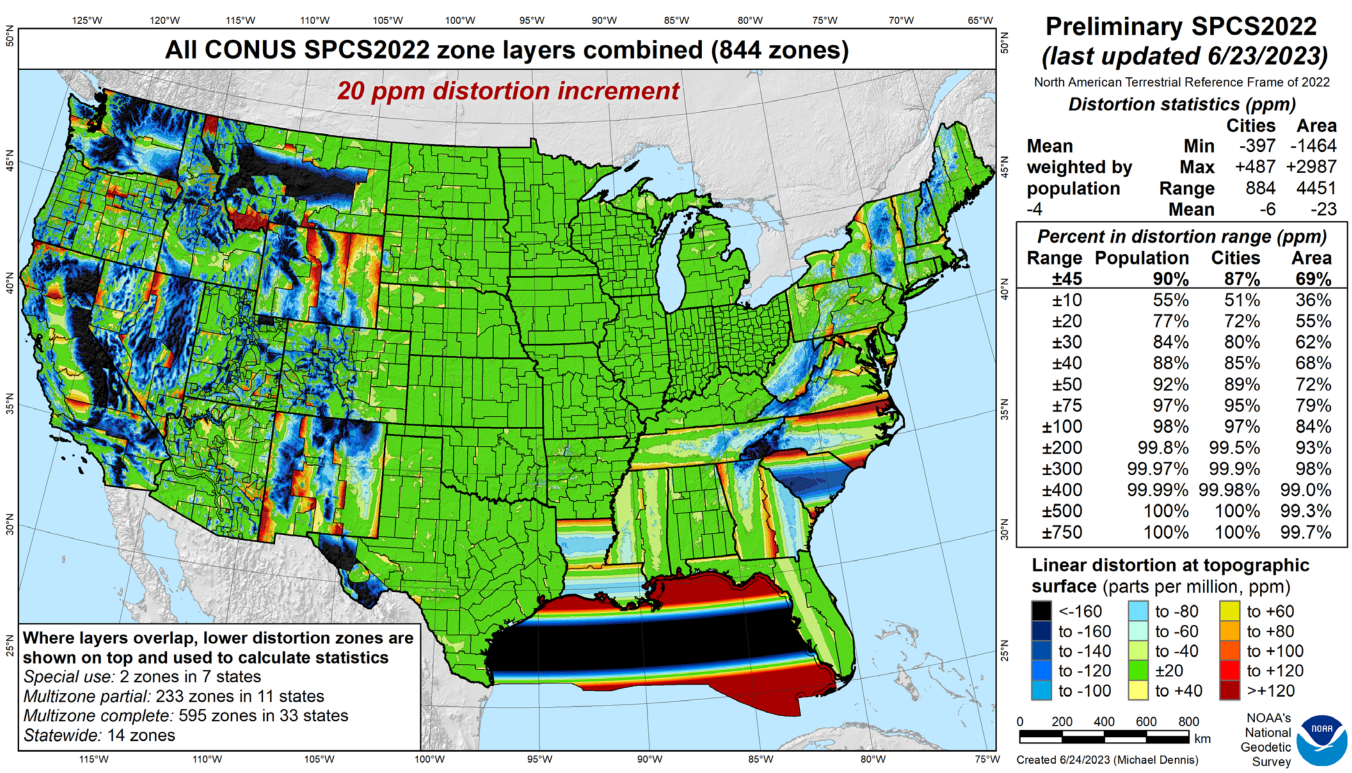
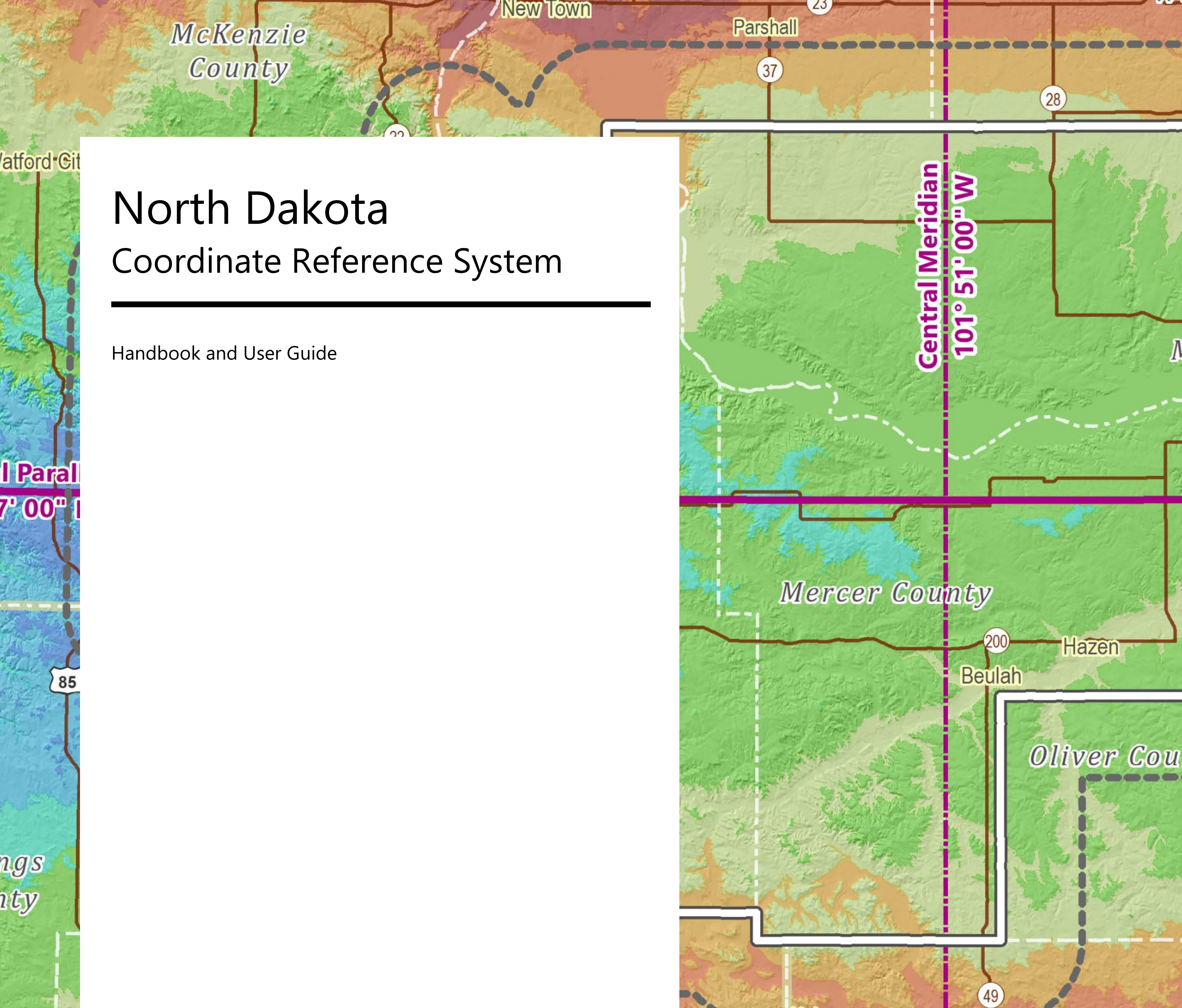


Figure 1. Linear distortion map for NDCRS Zones with statewide statistics.

Diagram 6: Map of low distortion zones throughout the country.





North Dakota
Coordinate Reference System

Handbook and User Guide

NORTH Dakota | Transportation
Be Legendary.

Version 1, February 2025

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Definition of Terms

Affine transformation – Linear mapping method that preserves points, straight lines and planes.

Conformal map projection – A projection where the linear distortion is unique (the same in every direction) at a point. The versions of this projection type used for SPCS are the Lambert Conformal Conic, Transverse Mercator, and Hotine Oblique Mercator.

Coordinate System – A reference system for a unique location of a point in space.

Datum – A basis for measurements over a limited area of the earth with its origin at a location on the Earth's surface. A reference surface that serves to provide known locations to begin surveys and create maps.

Geoid – An approximation of the earth's surface that is approximately equal to mean sea level extended over and under land areas.

Linear distortion – For conformal map projections, it is the amount by which a distance or length in a projected coordinate reference system from the actual distance on or near the topographical surface of the Earth. Also know as "scale error" when evaluated with respect to distances on the reference ellipsoid surface.

Low Distortion Projection – A map projection that minimizes linear distortion.

Map Projection – The representation of the earth's sphere onto a plane surface.

Raster – A grid of individual pixels that compose an image.

Reference Ellipsoid – An oblate ellipsoid of revolution that approximates the size and shape of the entire Earth geoid (mean sea level) or a large portion of it. When oriented with respect to a geometric reference frame or datum, it defines the reference surface for projected coordinate reference systems.

State plane grid – Plane coordinate system with perpendicular north-south and east-west lines

Topographic surface – The arrangement of physical features on land surfaces.

Vector – Comprised of paths defined by a start and end point on a Cartesian plane.

Zone centroid – Projection of a region of the Earth's surface to sea level at geographical center.

Abbreviations

3DEP – USGS 3D Elevation Program

CAD – Computer Aided Design

DEM - Digital Elevation Model

EPSG – European Petroleum Survey Group

GRS 80 – Reference Ellipsoid

GIS – Geographic Information System

GML – Geographic Markup Language

GPS – Global Positioning System

ift – International foot

LCC – Lambert Conformal Conic

LDP – Low Distortion Projection

m – meter

NAD 27 – North American Datum of 1927

NAD 83 – North American Datum of 1983

NATRF 2022 – North American Terrestrial Reference Frame of 2022

NAVD 88 - North American Vertical Datum of 1988

NDCRS – North Dakota Coordinate Reference System

NDDOT – North Dakota Department of Transportation

NGS – National Geodetic Survey

NOAA – National Oceanic and Atmospheric Administration

OM – Oblique Mercator

ppm – parts per million

Prj – Esri projection files

SPCS 1983 – State Plane Coordinate System 1983

SPCS 2022 – State Plane Coordinate System 2022

TM – Transverse Mercator

TRF – Terrestrial Reference Frame

WKT – Well Know Text

Introduction

Project Background

In April 2018 the Department of Commerce advertised action of notice to a proposed change to the State Plane Coordinate system. Under direction from the Department of Commerce, National Oceanic Atmospheric Administration's National Geodetic Survey will establish the State Plane Coordinate System of 2022 which supersedes previous coordinate systems. NGS allowed stakeholders to create a Low Distortion Projection coordinate system to be included as part of the SPCS 2022.

The purpose of this project is to design a multi-zone LDP coordinate system to provide coverage for North Dakota. Currently the North Dakota Department of Transportation uses State Plane Coordinate system of 83 as the basis for all state and federal highway projects. North Dakota has 53 counties spanning across an area approximately 350 miles east to west and 210 miles north to south. To achieve lower ground distortion state plane grid coordinates are projected to a ground coordinate using a combination factor created for each county.

The objective of the LDP coordinate system is to provide coverage over the entire state with as few zones as possible to achieve ±20 ppm (1:50,000) distortion for 95% of the state. Analysis and design of the LDPs was performed with software and digital topographic height models with design alternatives limited to Transverse Mercator and Lambert Conformal Conic projections. Design parameters were defined such that all zones of the final selected coordinate system are compatible with a wide range of commonly used commercial surveying, engineering, and GIS software.

NGS will create one statewide zone designed to a minimum of ±50 ppm distortion for SPCS 2022. To meet current and future surveying and design needs, having an LDP coordinate system referenced to the topographic surface will reduce projection distortion for surveying and mapping projects and eliminate the need to modify the SPCS with a combination factor. Together with the ND Society of Professional Land Surveyors, ND Department of Water Resources and ND State Geospatial Committee the NDDOT led the effort to design a multi-zone LDP layer to be a part of North Dakota's SPCS 2022.

Project Justification

The current NDDOT method to create project coordinates is to scale SPCS 83 grid coordinates to projected ground coordinates on the topographical surface. This approach reduces linear distortion at ground level but it does not optimally minimize distortion for projects with a large extent. NGS published their policies and procedures for the development of the SPCS 2022 including a provision to allow states to design zones with linear distortion below the ±50 ppm (1:20,000) criterion that NGS uses for SPCS 2022. The NDDOT opted to design a new LDP coordinate system for inclusion in SPCS 2022. Comprised of 16 LDP zones the new system has the following advantages over the previous scaled approach with SPCS 83:

- Optimally minimizes linear distortion over the largest areas possible so that ground coordinates can be obtained without additional scaling or modification.
- Simplifies data management by reducing the number of coordinate systems to 16 LDP zones.

- Facilitates data transferability and management with LDPs that are based on rigorously defined projected coordinate systems supported across a broad range of geospatial platforms.
- Simplifies surveying workflows by eliminating the need to perform vendor-specific methods of scaling, rotating, and translating spatial data including horizontal calibrations or localization.
- Generates coordinates that are distinct from previous coordinate systems and zone-specific coordinates that are easily identifiable.

Project Deliverables

Deliverables for this project consist of:

- Distortion maps: 11" x 17" distortion maps for each LDP zones and one map of the entire state showing all LDP zones. 8.5" x 11" index map of the state. These maps are available in PDF and PNG format as well as included as figures in this report.
- ArcMap V10.7 map packages files corresponding to all maps.
- GIS raster datasets of linear distortion, topographic height, geoid height, and hillshade, in IMG format and referenced to NAD 83 (2011) epoch 2010.00. The linear distortion rasters are in units of parts per million. The NDCRS distortion rasters are at 3 arc-second resolution for each zone and the entire state. Metadata (ISO 19139 compliant) also include for each raster.
- GIS vector feature dataset corresponding to the ArcMap documents and consisting of Esri shapefiles, referenced to NAD 83 (2011) epoch 2010.00. Includes NDCRS distortion contours, zone polygons projection axes, and zone centroid points. Metadata (ISO 19139 compliant) also included for each feature class.
- Esri projection (*.prj) files for each of the 16 zones of the final NDCRS definitions.
- Microsoft PowerPoint presentation to be used for training sessions. The presentation includes the project background, details about map projections, distortion maps, performance statistics, and zone parameters.
- Microsoft Excel spreadsheets of NDCRS projection parameters and centroid coordinates.

The North Dakota Coordinate Reference System

Design Criteria and Parameters

The following criteria were used to design the North Dakota Coordinate Reference System:

1. **Design Criteria:** The target accuracy was for linear distortion to be within ±20 ppm (±0.1 ft per mile or 1:50,000) for 95% of the state. Linear distortion is the difference in distance between a pair of projected grid coordinates as compared to the actual horizontal distance on the topographical ground surface of the Earth.
2. **Projection Types:** The 3 commonly used conformal projection types are allowed by NGS for inclusion in the SPCS 2022 include Lambert Conformal Conic, Transverse Mercator, and Oblique Mercator. The NDCRS designs were limited to LCC and TM projections. It should be noted that conformal projections are ideal because linear distortion is the same in every direction from a point. Review NGS procedures document NGS 2023-1214-03-A3 for technical specifications for SPCS 2022 designs.
3. **Geodetic Reference System:** NAD 83 is the *defining* datum for the NDCRS. SPCS 2022 will use the NATRF 2022 datum. Both datums use the Geodetic Reference System of 1980 as the reference ellipsoid.
4. **Zone Configuration:** The NDCRS zones were created to minimize the number of zones while achieving NGS's design criteria for linear distortion with the following criteria:
    a. Each zone consists of aggregated counties bounded by county lines.
    b. The total number of zones is 16, compiled from 53 counties with no zone overlap.
    c. Zones are numbered from left to right, top to bottom.
    d. Zones are named after the largest populated city in the zone based on recent Census information.
5. **Uniqueness:** NGS policy for SPCS 2022 zones specifies each zone must have a unique combination of origin latitude, longitude, and projection axis that is not repeated. The policy also specifies that all zones must differ by at least 10,000 meters horizontally from previous versions of SPCS, UTM, and other SPCS 2022 zone layers covering the same geographic region. Following NGS policy NDCRS was designed so the number values differ substantially from those of the following coordinate systems:
    a. State Plane Coordinate System of 2022, North Dakota Zone
    b. State Plane Coordinate System of 1983, North Dakota North and South zones, meters
    c. State Plane Coordinate System of 1927, North Dakota North and South zones, meters
    d. Universal Transverse Mercator of 1983, Zone 13 North, meters
    e. Universal Transverse Mercator of 1983, Zone 14 North, meters
6. **Linear Unit:** Originally the NGS required the meter to be the defining unit for SPCS2022. However, through feedback and analysis, NGS now requires the international foot as the defining unit for false northings and eastings and it is also the intended working unit of the projected coordinates. This document was created between policy decisions and references both meter and international feet.

7. **Projected coordinate values:** NDCRS coordinates were specified that northings and eastings are not equal everywhere within a zone and coordinate values are always positive in a zone. Specifically:
   a. Easting coordinates are unique for each zone.
   b. Northings are not unique for each zone. TM projections have a value of zero. Northings for LCC projections are the same based on whether the zone falls in the north or south part of the state.

Table 1 shows the final design parameters for North Dakota's 16 zones including number of counties in each zone, zone number and zone code for quick reference. All projections are referenced to NAD 83 as the geodetic datum. Six of the 16 zones are TM projections with a false northing of zero at their origin latitude. The origin latitude for TM zones 1-3 and 6 is 46°30'00"N and 45°45'00"N for zones 15 and 16. All of the other ten zones are one-parallel LCC projections where the origin latitude is also the standard parallel.

Table 2 provides a cross-reference showing each of the 53 North Dakota counties and corresponding zone.

Figure 1 shows linear map projection distortion at the topographical surface for all 16 zones and overall statewide performance statistics. The goal of 95% of the state having ±20 ppm distortion was achieved and 63% has distortion with ±10 ppm while 99% of the state is within ±30 ppm. Mean distortion is only slightly negative, -1.0 ppm attributed to large, broad high areas notable in the Turtle Mountains.

Figure 2 graphically shows each zone centroid to illustrate the expected magnitude and difference between NDCRS coordinates in each zone. Zone centroids are expressed in meters and when converted to international feet the beginning of the easting coordinate is equivalent to the zone number.

Appendix A provides distortion maps for each of the 16 zones showing projection parameters and distortion at a 6-mile buffer around the entire zone. These maps will be available on the NDDOT website for printing on 11" x 17" tabloid paper at the scale at which they were created.

Appendix B provides distortion and ellipsoid height values for incorporated places in Table B1.

Appendix C Equations for computing projection grid point scale factors.

Table 1. Defining parameters for the 16 zones of the NDCRS – International feet

- All zones are referenced to the North American Datum of 1983 and based on the Geodetic Reference System of 1980 ellipsoid.
- Projection types: “TM” = Transverse Mercator; “LC” = Lambert Conformal Conic.
- For LC projections, the origin latitude is the standard parallel or projection axis.

Zone Number	Zone Code	Zone Abrev.	Zone Name	Number of Counties	Projection Type	Origin Latitude		Origin Longitude		Projection Axis Scale	False Northing (ift)	False Easting (ift)	False Northing (m)	False Easting (m)
						deg-min	decimal degree	deg-min	decimal degree					
1	381001	ND_WLS	Williston	3	TM	46° 30' N	46.5000°	103° 27' W	-103.4500°	1.000 092	0	1,500,000	0	457,200
2	381002	ND_NT	New Town	2	TM	46° 30' N	46.5000°	102° 27' W	-102.4500°	1.000 094	0	2,500,000	0	762,000
3	381003	ND_MNT	Minot	2	TM	46° 30' N	46.5000°	101° 27' W	-101.4500°	1.000 080	0	3,625,000	0	1,104,900
4	381004	ND_BOT	Bottineau	4	LC	48° 36' N	48.6000°	99° 42' W	-99.7000°	1.000 066	500,000	4,500,000	152,400	1,371,600
5	381005	ND_DL	Devils Lake	5	LC	48° 09' N	48.1500°	99° 27' W	-99.4500°	1.000 056	500,000	5,500,000	152,400	1,676,400
6	381006	ND_GF	Grand Forks	5	TM	46° 30' N	46.5000°	97° 24' W	-97.4000°	1.000 034	0	6,500,000	0	1,981,200
7	381007	ND_BEU	Beulah	3	LC	47° 27' N	47.4500°	101° 51' W	-101.8500°	1.000 090	500,000	7,500,000	152,400	2,286,000
8	381008	ND_CAR	Carrington	5	LC	47° 33' N	47.5500°	99° 18' W	-99.3000°	1.000 072	500,000	8,500,000	152,400	2,590,800
9	381009	ND_DKS	Dickinson	3	LC	47° 00' N	47.0000°	103° 03' W	-103.0500°	1.000 112	375,000	9,500,000	114,300	2,895,600
10	381010	ND_BIS	Bismarck	4	LC	46° 48' N	46.8000°	100° 45' W	-100.7500°	1.000 081	375,000	10,500,000	114,300	3,200,400
11	381011	ND_JMT	Jamestown	2	LC	46° 57' N	46.9500°	98° 36' W	-98.6000°	1.000 067	375,000	11,500,000	114,300	3,505,200
12	381012	ND_BOW	Bowman	4	LC	46° 18' N	46.3000°	103° 00' W	-103.0000°	1.000 124	375,000	12,500,000	114,300	3,810,000
13	381013	ND_CB	Cannon Ball	2	LC	46° 18' N	46.3000°	101° 18' W	-101.3000°	1.000 093	375,000	13,500,000	114,300	4,114,800
14	381014	ND_LIN	Linton	3	LC	46° 18' N	46.3000°	99° 51' W	-99.8500°	1.000 082	375,000	14,375,000	114,300	4,381,500
15	381015	ND_OAK	Oakes	2	TM	45° 45' N	45.7500°	98° 18'W	-98.3000°	1.000 061	0	15,375,000	0	4,686,300
16	381016	ND_FAR	Fargo	4	TM	45° 45' N	45.7500°	97° 12'W	-97.2000°	1.000 032	0	16,375,000	0	4,991,100

Table 2. North Dakota Counties and corresponding NDCRS zone.

County	Zone	Zone No.	County	Zone	Zone No.	County	Zone	Zone No.	County	Zone	Zone No.
Adams	Bowman	12	Emmons	Linton	14	McLean	Beulah	7	Sargent	Fargo	16
Barnes	Jamestown	11	Foster	Carrington	8	Mercer	Beulah	7	Sheridan	Carrington	8
Benson	Devils Lake	5	Golden Valley	Dickinson	9	Morton	Bismarck	10	Sioux	Cannon Ball	13
Billings	Dickinson	9	Grand Forks	Grand Forks	6	Mountrail	New Town	2	Slope	Bowman	12
Bottineau	Bottineau	4	Grant	Cannon Ball	13	Nelson	Devils Lake	5	Stark	Dickinson	9
Bowman	Bowman	12	Griggs	Carrington	8	Oliver	Bismarck	10	Steele	Grand Forks	6
Burke	New Town	2	Hettinger	Bowman	12	Pembina	Grand Forks	6	Stutsman	Jamestown	11
Burleigh	Bismarck	10	Kidder	Bismarck	10	Pierce	Devils Lake	5	Towner	Bottineau	4
Cass	Fargo	16	LaMoure	Oakes	15	Ramsey	Devils Lake	5	Traill	Grand Forks	6
Cavalier	Bottineau	4	Logan	Linton	14	Ransom	Fargo	16	Walsh	Grand Forks	6
Dickey	Oakes	15	McHenry	Devils Lake	5	Renville	Minot	3	Ward	Minot	3
Divide	Williston	1	McIntosh	Linton	14	Richland	Fargo	16	Wells	Carrington	8
Dunn	Beulah	7	McKenzie	Williston	1	Rolette	Bottineau	4	Williams	Williston	1
Eddy	Carrington	8									

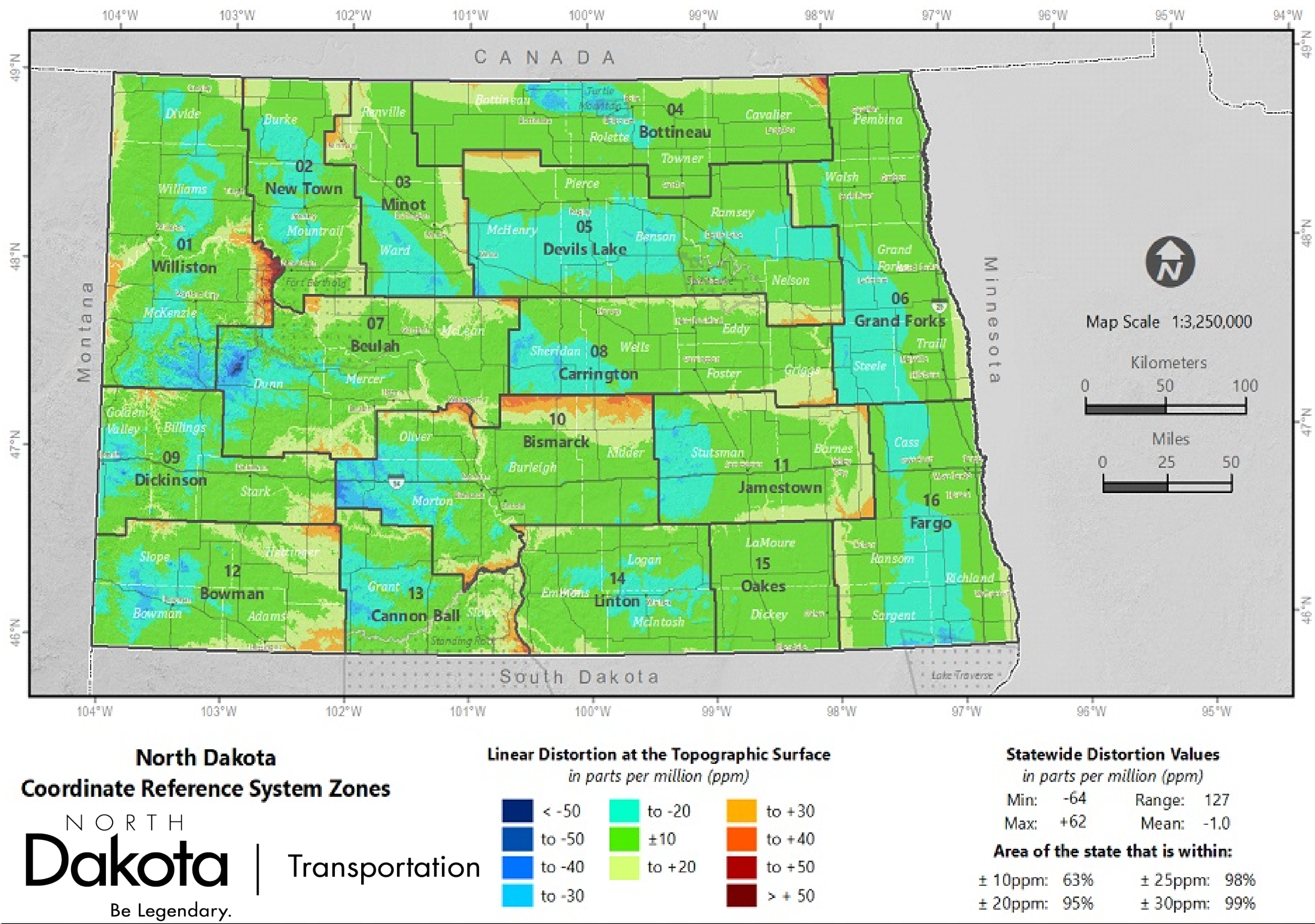

Figure 1. Linear distortion map for NDCRS Zones with statewide statistics.

Projected Coordinate Values

To illustrate the expected magnitude and difference between NDCRS coordinates in each zone, projected coordinates for the centroid of each zone are provided in Figure 2. It should be noted that the centroid coordinates have been rounded to exactly the nearest whole arc-second divisible by 3. This provides an exact decimal degree number to 4 decimal places of precision to serve as a check on NDCRS coordinate computations in computer software. This data is provided in Table 3 which includes the ellipsoid height rounded to the nearest whole meter. For additional computation checks, Table 4 provides the NDCRS coordinate values for each zone centroid point in both meters and international feet along with the linear distortion in ppm, the combined grid scale factor, and convergence angle. When converted to feet, the beginning of the easting coordinate value is equivalent to the zone number.

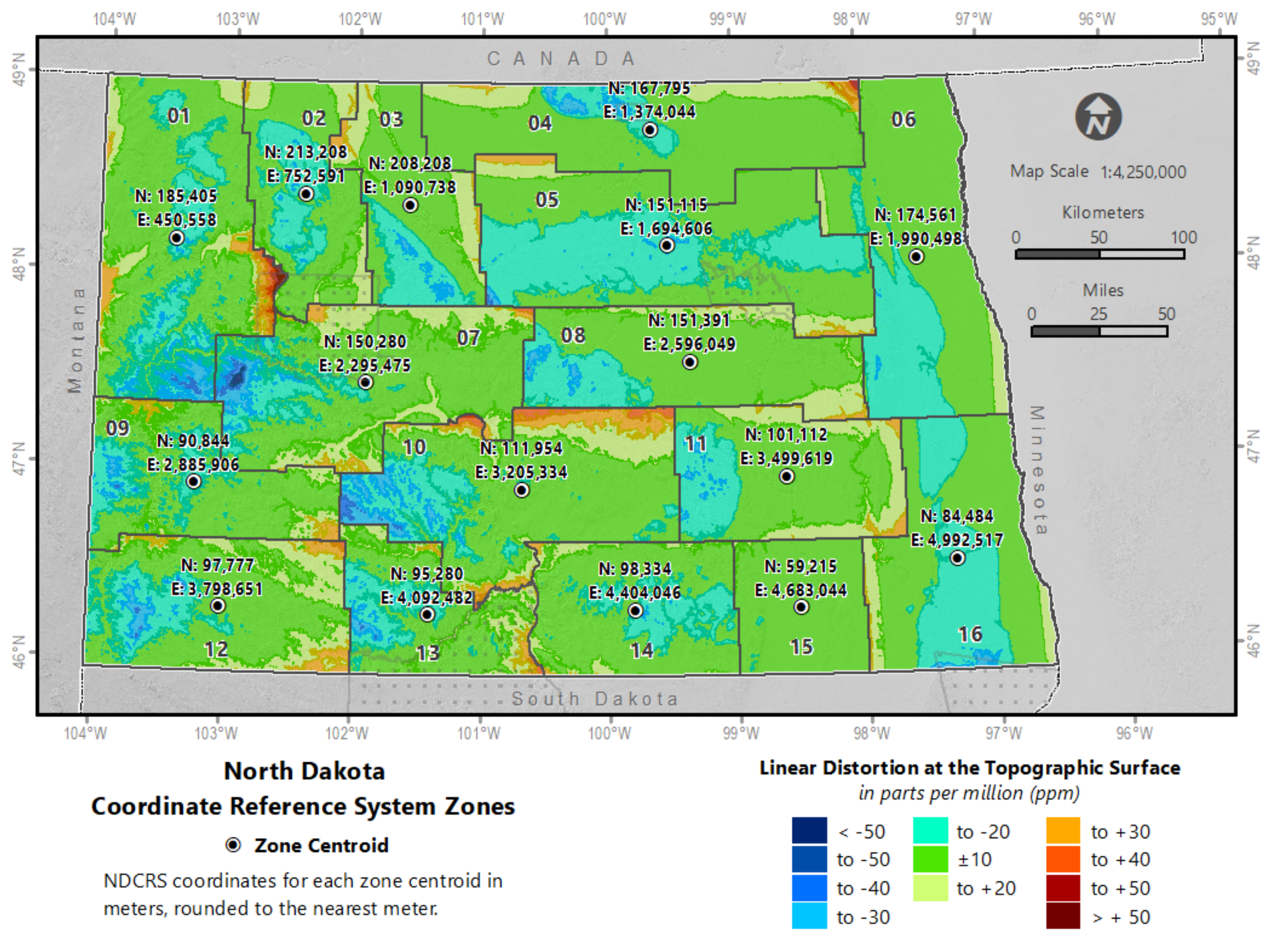

Figure 2. Projected centroid coordinates for NDCRS zones (rounded to nearest meter)

Table 3. Zone centroid coordinates. Latitude and longitude values are to the exact whole arc second and ellipsoid heights are to the whole meter.

Zone	Zone Name	degrees/minutes/seconds			decimal degrees			Ellipsoid Height (m)	Northing (m)	Easting (m)	Northing (ift)	Easting (ift)
		Latitude	Longitude		Latitude	Longitude						
			East	West		East	West					
1	Williston	48° 10' 03'' N	256° 33' 27'' E	103° 26' 33'' W	48.1675	256.5575	-103.4425	654	185,405.30	450,557.93	608,285.11	1,478,208.42
2	New Town	48° 25' 03'' N	257° 35' 06'' E	102° 24' 54'' W	48.4175	257.585	-102.415	697	213,207.85	752,590.99	699,500.83	2,469,130.54
3	Minot	48° 22' 21'' N	258° 25' 30'' E	101° 34' 30'' W	48.3725	258.425	-101.575	484	208,207.52	1,090,738.44	683,095.53	3,578,538.19
4	Bottineau	48° 45' 36'' N	260° 21' 18'' E	99° 38' 42'' W	48.76	260.355	-99.645	515	167,795.22	1,374,044.03	550,509.25	4,508,018.48
5	Devils Lake	48° 09' 36'' N	260° 28' 39'' E	99° 31' 21'' W	48.16	260.4775	-99.5225	475	151,114.54	1,694,606.11	495,782.60	5,559,731.34
6	Grand Forks	48° 04' 12'' N	262° 28' 21'' E	97° 31' 39'' W	48.07	262.4725	-97.5275	254	174,560.74	1,990,497.82	572,705.84	6,530,504.67
7	Beulah	47° 27' 09'' N	258° 05' 24'' E	101° 54' 36'' W	47.4525	258.09	-101.91	564	150,279.72	2,295,474.90	493,043.70	7,531,085.64
8	Carrington	47° 33' 45'' N	260° 38' 51'' E	99° 21' 09'' W	47.5625	260.6475	-99.3525	456	151,391.21	2,596,048.88	496,690.31	8,517,220.72
9	Dickinson	46° 55' 03'' N	256° 45' 54'' E	103° 14' 06'' W	46.9175	256.765	-103.235	790	90,844.08	2,885,906.42	298,044.89	9,468,196.91
10	Bismarck	46° 54' 27'' N	259° 19' 12'' E	100° 40' 48'' W	46.9075	259.32	-100.68	561	111,953.91	3,205,333.54	367,302.85	10,516,186.16
11	Jamestown	46° 57' 36'' N	261° 23' 42'' E	98° 36' 18'' W	46.96	261.395	-98.605	430	101,111.79	3,499,619.41	331,731.58	11,481,690.98
12	Bowman	46° 16' 48'' N	256° 58' 57'' E	103° 01' 03'' W	46.28	256.9825	-103.0175	819	97,776.73	3,798,651.08	320,789.81	12,462,766.01
13	Cannon Ball	46° 15' 27'' N	258° 36' 09'' E	101° 23' 51'' W	46.2575	258.6025	-101.3975	643	95,280.02	4,092,481.74	312,598.50	13,426,777.37
14	Linton	46° 17' 06'' N	260° 12' 09'' E	99° 47' 51'' W	46.285	260.2025	-99.7975	594	98,333.85	4,404,046.22	322,617.61	14,448,970.55
15	Oakes	46° 16' 57'' N	261° 28' 48'' E	98° 31' 12'' W	46.2825	261.48	-98.52	427	59,215.38	4,683,043.98	194,276.20	15,364,317.52
16	Fargo	46° 30' 36'' N	262° 42' 09'' E	97° 17' 51'' W	46.51	262.7025	-97.2975	292	84,484.25	4,992,516.78	277,179.31	16,379,648.23

Table 4. Computed values at zone centroids. These values are provided for computation checks.

Zone	Zone Name	Northing (m)	Easting (m)	Northing (ift)	Easting (ift)	Linear Distortion (ppm)	Combined Factor	Convergence angle (DMS)
1	Williston	185,405.3030	450,557.9273	608,285.1149	1,478,208.4230	-10.4955	0.999 989 5045	+0°00'20.12"
2	New Town	213,207.8539	752,590.9874	699,500.8332	2,469,130.5361	-15.1524	0.999 984 8476	+0°01'34.25"
3	Minot	208,207.5187	1,090,738.4403	683,095.5337	3,578,538.1900	5.1982	1.000 005 1982	–0°05'36.37"
4	Bottineau	167,795.2187	1,374,044.0316	550,509.2478	4,508,018.4763	-10.8166	0.999 989 1834	+0°02'28.52"
5	Devils Lake	151,114.5377	1,694,606.1110	495,782.6039	5,559,731.3353	-18.4295	0.999 981 5705	–0°03'14.42"
6	Grand Forks	174,560.7386	1,990,497.8237	572,705.8355	6,530,504.6711	-4.7003	0.999 995 2997	–0°05'41.48"
7	Beulah	150,279.7196	2,295,474.9034	493,043.6994	7,531,085.6409	1.5986	1.000 001 5986	–0°02'39.12"
8	Carrington	151,391.2068	2,596,048.8752	496,690.3111	8,517,220.7191	0.5505	1.000 000 5505	–0°02'19.46"
9	Dickinson	90,844.0822	2,885,906.4171	298,044.8890	9,468,196.9065	-10.7992	0.999 989 2008	–0°08'07.08"
10	Bismarck	111,953.9088	3,205,333.5418	367,302.8503	10,516,186.1609	-5.1812	0.999 994 8188	+0°03'03.70"
11	Jamestown	101,111.7863	3,499,619.4108	331,731.5822	11,481,690.9803	-0.3875	0.999 999 6125	–0°00'13.15"
12	Bowman	97,776.7336	3,798,651.0791	320,789.8083	12,462,766.0076	-4.3276	0.999 995 6724	–0°00'45.55"
13	Cannon Ball	95,280.0225	4,092,481.7412	312,598.4991	13,426,777.3662	-7.5239	0.999 992 4761	–0°04'13.76"
14	Linton	98,333.8480	4,404,046.2242	322,617.6115	14,448,970.5519	-11.0820	0.999 988 9180	+0°02'16.64"
15	Oakes	59,215.3846	4,683,043.9796	194,276.1962	15,364,317.5184	-2.4051	0.999 997 5949	–0°09'32.42"
16	Fargo	84,484.2543	4,992,516.7801	277,179.3119	16,379,648.2288	-13.0849	0.999 986 9151	–0°04'14.65"

Using the NDCRS requires that its parameters be entered into software by the software user or the manufacturer. The possibility exists for system parameters to be entered incorrectly and not all software is programmed to perform calculations the same way resulting in incorrect results. For this reason, computation checks should be done to ensure correctness. If the latitude and longitude values for a zone listed in Table 3 are entered into software the results should show the NDCRS values shown. Conversely, if the NDCRS coordinates are entered they should produce the latitude and longitude values shown. If the software does not produce the value shown in Table 3, check to ensure all parameters have been entered correctly, the units are correct, and the datum has been properly defined with the correct reference ellipsoid and no datum transformation. Datum transformations are a common cause of coordinate computation errors. To duplicate the coordinates in Table 3, the latitude and longitude as shown must be used in the projection algorithms and cannot be transformed before projecting. The reverse is also true – the latitude and longitude generated by de-projecting the NDCRS coordinates must not be transformed after de-projection.

The NDCRS projected coordinates are defined such that they differ from other existing coordinate systems defined in North Dakota. These systems include State Plane Coordinate System of 2022 North Dakota statewide zone, State Plane North Dakota both North and South zones based in both NAD 83 defined by meters and NAD 27 defined by US survey foot; UTM 83 zones 13 North and 14 North defined by meters. To illustrate, the horizontal difference was calculated from the zone centroids using the NDCRS coordinates listed in table 3 and 4 to the various other coordinate systems previously identified and shown in Table 5.

Table 5. Differences in coordinate values between NDCRS and other North Dakota coordinate systems.

Differences are given in meters, rounded to the nearest whole meter.

NDCRS Zone No.	NDCRS Zone Name	SPCS 27 and 83 Zone	UTM Zone	Horizontal difference from NDCRS (meters)		
				SPCS 27	SPCS 83	UTM
1	Williston	N	13	78,820	86,345	5,153,335
2	New Town	N	13	289,737	299,205	5,153,091
3	Minot	N	14	563,439	573,026	5,211,616
4	Bottineau	N	14	702,115	711,744	5,313,686
5	Devils Lake	N	14	1,012,489	1,022,122	5,327,848
6	Grand Forks	N	14	1,160,543	1,170,169	5,332,446
7	Beulah	S	14	1,792,841	1,802,470	5,492,197
8	Carrington	N	14	1,902,112	1,911,736	5,539,158
9	Dickinson	S	13	2,485,094	2,494,723	5,580,981
10	Bismarck	S	14	2,609,543	2,619,175	5,820,444
11	Jamestown	S	14	2,746,132	2,755,766	5,901,279
12	Bowman	S	13	3,383,115	3,392,746	5,932,165
13	Cannon Ball	S	14	3,552,168	3,561,799	6,290,495
14	Linton	S	14	3,740,391	3,750,025	6,403,334
15	Oakes	S	14	3,920,848	3,930,484	6,546,571
16	Fargo	S	14	4,137,196	4,146,834	6,686,343

NDCRS Design Methodology and Results

NDCRS design was completed using a digital elevation model derived from the United States Geological Survey's 3D Elevation Program at a resolution of 1 arc-second which translates to approximately 300 ft on the ground. The 3DEP provides the North American Vertical Datum of Datum of 1988 orthometric heights. The NAVD 88 heights were converted to NAD 83 ellipsoid heights by adding the NGS hybrid geoid model GEOID18 interpolated to the same resolution as the DEM.

Map projection linear distortion is caused by change in topographic height and Earth curvature due to the projected surface departing from the topographic surface. Total distortion is based on height and Earth curvature making it near impossible to have a singular design height for large areas. The reason this is particularly true for large areas is that the contribution of Earth curvature to total distortion increases as the size of the area increases. Consequently, topographic ellipsoid height was not used directly as a design parameter.

A ±400-foot change in height causes approximately ±20 ppm of distortion. Figure 3 shows the topographic ellipsoid heights for North Dakota. The state has a range in topographic height of about 2,800 feet. This range in topographic height alone would require at least four zones to keep distortion within ±20 ppm. However, curvature of the Earth also needs to be taken into consideration where a zone width of 70 miles perpendicular to the projection axis will cause about ±20 ppm, requiring North Dakota's east-west length of approximately 350 miles to create about six zones. By combining topographic height and curvature of the earth it would suggest that about 20 zones are needed to achieve ±20 ppm distortion across North Dakota. In keeping with the project goal to have zones follow county boundaries while achieving the fewest number of zones possible, the NDCRS was developed with 16 zones.

Design of the NDCRS:

1) A review was completed of linear distortion by county for SPCS 83 using the NDDOT county coordinate conversion factors. Preliminary zones by county groupings was created taking into consideration large population centers and transportation corridors. The preliminary design included 15 zones. Initial design and analyses was performed on these zones, resulting in four of the zones not achieving the objective of 95% of the zone having ±20 ppm distortion. Alternative zones were created for these areas along with options for the east and west ends of the state. These design alternatives were evaluated resulting in further refinement in the next design phase.
2) Based on initial feedback and detailed review of preliminary designs, 16 final zones were established across the state. Projection determination depends on topographical height and slope. Zones that are longer north and south have less distortion using a Transverse Mercator projection while zones that are longer east and west perform better with a Lambert Conformal Conic projection. The final design included six TM zones and ten LCC zones. Design and analyses were based on a one-kilometer grid of 183,096 ellipsoid height points across the state, 495 city and town points, and 26,548 population points based on the block centroids from the 2010 decennial Census.
3) Final design maps are in Appendix A. A 6-mile buffer extending past the zone boundaries is included to provide visual guidance for projects that occur near or cross over zone boundaries that might benefit from lower distortion using the projection from an adjacent zone.

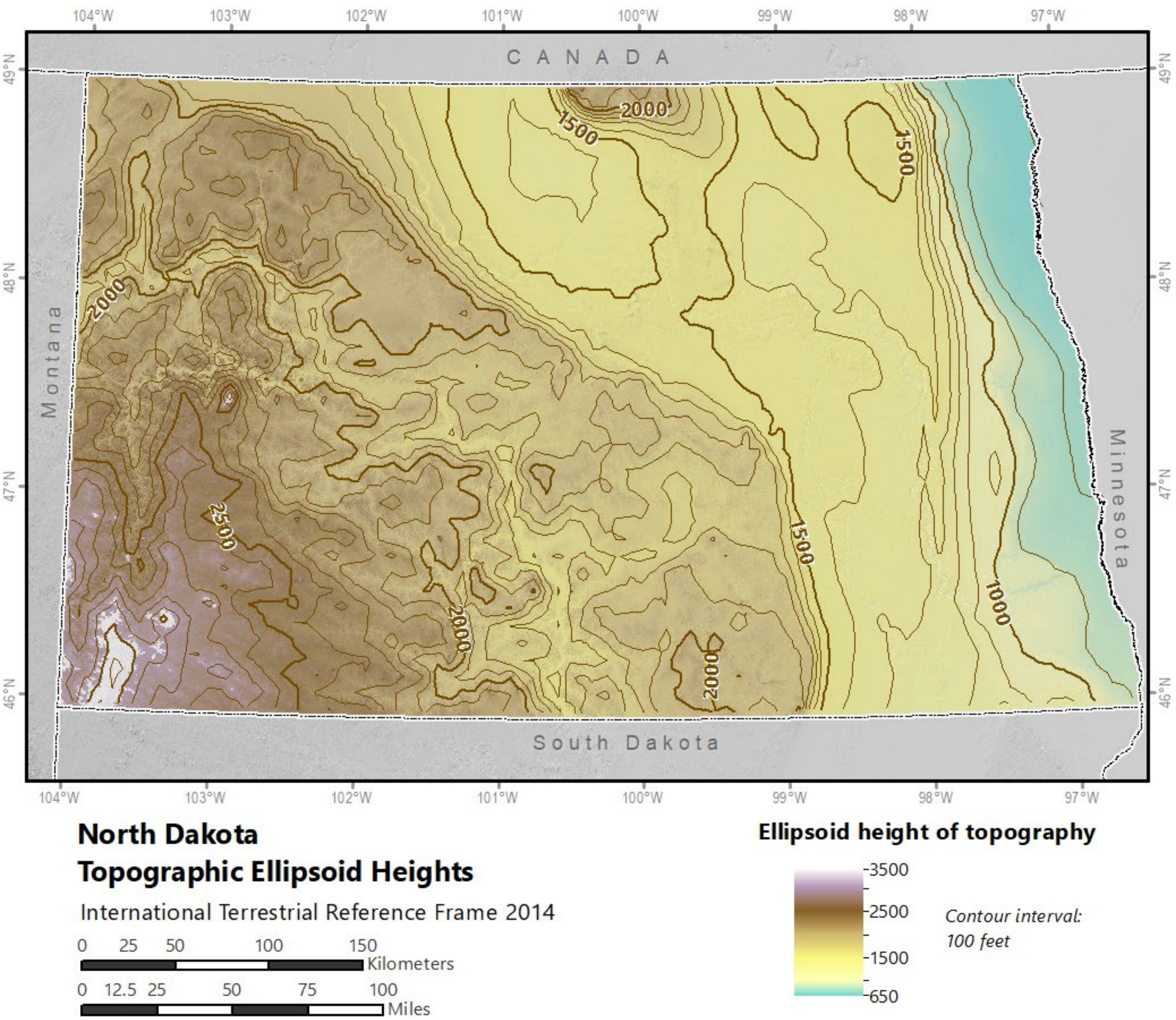

Figure 3. Topographic Ellipsoid Heights in North Dakota

Final NDCRS distortion and ellipsoid height statistics for each zone and the entire state are given in Table 6. The distortion and ellipsoid height statistics were computed from the 3 arc-second distortion and ellipsoid rasters. Table 6 gives distortion statistics for minimum, maximum, range, mean, standard deviation and ellipsoid height. Slightly over 95% of the state has distortion within ±20 ppm which meets the stated objective. Five of the 16 zones, Williston (91.0%), Beulah (93.4%), Bismarck (85.1%), Bowman (92.5%), and Cannon Ball (94.5%) have less than 95% of the zone with ±20 ppm. The smallest zone, Oakes, has complete coverage with distortion at ±15 ppm. Grand Forks, among one of the largest zones, is almost completely (99.9%) within ±20 ppm. Half of the zones have complete coverage at ±30 ppm.

The highest distortion values in the state occur in the Williston (+62.4 ppm) and Beulah (-64.4 ppm). In Williston this occurs along the Missouri River at the far east edge of the zone. In Beulah it is the Killdeer Mountains along the western edge of the zone causing the most distortion. Adjacent zones provide lower distortion in these areas if needed.

Table 6 shows the ellipsoid height range of 2,770 ft. from 655 ft to 3,425 ft. The range for each zone vary and the highest range occurs in Beulah at 1,697 ft due to the Killdeer Mountains.

Table 6. Linear distortion and ellipsoid height statistics for each zone and statewide.

Ellipsoid heights are shown in international feet. Table 3 provides ellipsoid height values in internationl feet.

Zone	Zone Name	Distortion in parts per million (ppm)					Percent of Zone within stated PPM			Ellipsoid Height (ift)		
		Min	Max	Range	Mean	St. Dev.	±10 ppm	±20 ppm	±30 ppm	Min	Max	Range
1	Williston	-35.0	62.4	97.4	-1.4	12.3	61.8	91.0	98.0	1,769	2,712	943
2	New Town	-23.3	22.9	46.2	-1.1	10.1	59.7	98.9	100.0	1,717	2,490	774
3	Minot	-24.8	29.0	53.8	0.5	11.2	57.5	96.2	100.0	1,417	2,294	877
4	Bottineau	-31.1	49.1	80.2	0.0	9.6	74.6	95.8	99.4	862	2,444	1,581
5	Devils Lake	-34.5	31.5	66.0	-5.9	9.1	53.2	97.3	99.9	1,194	2,101	907
6	Grand Forks	-21.1	21.5	42.6	-3.5	7.6	71.1	99.9	100.0	655	1,569	915
7	Beulah	-64.4	40.8	105.1	1.9	11.4	61.2	93.4	98.9	1,534	3,231	1,697
8	Carrington	-31.7	30.1	61.8	1.0	10.0	65.7	97.6	100.0	1,181	2,172	991
9	Dickinson	-46.3	31.9	78.2	-1.0	9.8	67.6	96.3	99.9	1,928	3,367	1,439
10	Bismarck	-37.7	42.7	80.4	1.0	13.5	57.0	85.1	96.6	1,488	2,485	997
11	Jamestown	-25.0	31.7	56.7	2.0	10.8	55.0	96.6	100.0	1,051	2,058	1,007
12	Bowman	-38.5	38.7	77.2	1.5	11.6	55.0	92.5	99.7	2,104	3,425	1,322
13	Cannon Ball	-34.5	38.9	73.4	0.6	11.4	56.0	94.5	99.4	1,531	2,758	1,227
14	Linton	-22.5	27.9	50.4	-2.1	8.3	72.6	99.1	100.0	1,524	2,193	669
15	Oakes	-14.0	13.6	27.5	1.0	3.9	99.0	100.0	100.0	1,188	2,161	973
16	Fargo	-40.8	22.9	63.7	-5.7	8.9	56.6	98.2	100.0	750	1,636	885
Statewide		**-64.4**	**62.4**	**126.8**	**-1.0**	**10.7**	**62.8**	**95.3**	**99.3**	**655**	**3,425**	**2,770**

Linear distortion and ellipsoid heights are also provided in Appendix B for 94 incorporated places in North Dakota with an estimated population of 500 or more based on Census 2019 population estimates. For locations not included in Table B-1**,** distortion can be estimated from the maps provided in Appendix A**.** If the distortion rasters are available distortion can be obtained at any location in the state using the 3 arc-second raster for the most accurate values. The table includes the coordinate of the point where the distortion is computed so they can also serve as a distortion computation check.

Using the NDCRS

Defining the NDCRS in Software

The NDCRS was designed using TM and LCC projections which are well defined existing conformal map projection types commonly available in GIS, surveying, and engineering software. Definitions for the NDCRS zones were provided in Esri projection format as deliverables for this project. The *.prj format is a Well-Known Text mark-up language representation of a projected coordinate system, as defined by the Open Geospatial Consortium in 2019. WKT for coordinate reference systems was developed to provide a standardized means of defining coordinate systems in machine-readable format.

Not all geospatial software directly uses WKT representations for defining projected coordinate systems. For software that is not included in the deliverable and that cannot consume the provided WKT representations the projection parameters in Table 1 can be used to manually define NDCRS zones.

In many coordinate system definition formats the latitude and longitude are represented in decimal degrees. When repeating decimal digits occur (ex. 97°20'W = -97.333333333333…°) it is recommended that full double-precision be used to represent the number. 16 digits should be used corresponding to 14 decimal places for latitude and longitude less than 100° and 13 decimal places for longitude greater than or equal to 100°. The WKT format should automatically use double precision representation when 16 or more significant digits are provided while defining the parameters. Ten decimal places correspond to about 0.000 04 ft or less which is likely sufficient for most practical applications. It is good practice to define the parameters to full double precision representation to help minimize problems with accumulated round-off error including repeatedly projecting and de-projecting coordinates.

An important part of defining the projection parameters is performing computation checks. Performing checks provides a means for capturing overlooked errors in the parameters and projection algorithms. The centroid coordinates in Table 3 are provided for that purpose. Geospatial software should match the projected coordinates to the precision shown to better than ±0.0001 ft. You can also check if the northings and eastings in Table 3 are de-projected, they should yield latitude and longitude values that matches those in the table to six decimal places (±0.000 001”).

In rare instances, the single-parallel LCC projections used for the NDCRS may be problematic to implement due to software limitations including surveying field software. For such situations, satisfactory alignment with the NDCRS in small areas can usually be achieved using a best-fit planar conformal transformation (affine transformations should not be used because the NDCRS itself is conformal). Such an approach is referred to horizontal calibration or localization in geospatial software. For areas with a maximum dimension of less than about 3 miles differential distortion distribution will usually cause less than 0.05 ft mean alignment error. In these cases the transformation should be based on a minimum of three common points distributed across the extent of the area. Common points are any physical points correctly referenced to NAD 83 that can be observed in the field and for which NDCRS coordinates are available.

Although such an approach can produce acceptable results it should only be applied if the rigorous projection definitions cannot be used and completed by practitioners with sufficient knowledge and skill in using and interpreting such transformations.

Distortion and Ground Distance Computations

Linear distortion, δ, at any point in a projected coordinate system can be computed using the following equation,

$$\delta = k\left(\frac{R}{R+h}\right) - 1$$

where k is the projection grid point scale factor

R is the geometric mean radius of curvature of the GRS-80 ellipsoid

h is the NAD 83 ellipsoid height

Distortion is multiplied by 1,000,000 to obtain parts per million (ppm). Note that if the minus 1 is removed from the equation it becomes the *combined (scale) factor* familiar to surveyors. The quantity in parentheses is the *height (elevation) factor*. The combined scale factor is given for the points listed in table 5.

For greater accuracy, R can be computed at any latitude as

$$R = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$$

where a is the semi-major axis of the GRS-80 ellipsoid = 6,378,137 m (exact) ≈ 20,925,646.3255 ift
e^2 is the first eccentricity squared of the GRS-80 ellipsoid = 0.0066943800229
φ is the geodetic latitude

Some software may not compute the grid point scale factor, k. Equations for computing k are fairly complex and are a function of position as well as the projection characteristics. These values are given in Appendix C for the TM and LCC projections.

Linear distortion is defined at a point, but practical applications are concerned with how well the projected grid distance matches the true horizontal ground distance between a pair of points. Such evaluations require an accurate method for computing ground distance. Although there is no standard definition for ground distance it can be argued that a reasonable definition is the curved distance parallel to the ellipsoid surface at the mean ellipsoid height of the endpoints. Two relatively simple methods for computing such distance based on this definition are provided below.

One method for computing horizontal ground distance, D, consists of scaling the ellipsoid distance (geodesic) using the average of the ellipsoid heights at the endpoints:

$$D = s\left(1 + \frac{\bar{h}}{R_m}\right)$$

where s is the ellipsoid distance (geodesic)
$\bar{h}$ is the average ellipsoid height of the two points
R_m is the geometric mean radius of curvature at the midpoint latitude of the two points

If software for computing the ellipsoid distance is not available, the NGS Geodetic Tool Kit inversing tools can be used at the following NGS webpage: geodesy.noaa.gov/TOOLS/Inv_Fwd/Inv_Fwd.html

A second method for computing a horizontal ground distance makes use of a GPS vector directly. Neglecting Earth curvature, this distance can be computed using the vector East-Centered, Earth-Fixed Cartesian coordinate deltas,

$$D = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2}$$

where ΔX, ΔY, ΔZ are the GNSS vector components as ECEF Cartesian coordinate deltas
Δh is the change in ellipsoid height between vector endpoints

This method can also be used with endpoint coordinates instead of a GNSS vector by converting the latitude, longitude, and ellipsoid heights to X, Y, Z ECEF coordinates and then using the difference in ECEF coordinates. If software is not available for computing ECEF coordinates, the NGS Geodetic Tool Kit *XYZ Conversion* tool can be used for this purpose following the NGS webpage: geodesy.noaa.gov/TOOLS/XYZ/xyz.shtml.

Curvature increases the horizontal ground distance but for distances less than 20 miles, the error is less than 1 part per million which translates to 0.1 ft. The straight-line horizontal distance can be multiplied by the following curvature correction factor C to get the approximate curved horizontal ground distance, where all variables are as defined previously.

$$C = \frac{2R_m \sin^{-1}(D/2R_m)}{D}$$

With the curvature correction, distances of less than 100 miles the error is less than 0.005 ppm, less than 0.003 ft. A mean Earth radius of curvature of 5,865,000 meters can be used for North Dakota or computed using the equation provided earlier.

Although equations have been provided here for accurately computing long ground distances it is important to understand that linear distortion varies continuously along the line between two points. When a ground distance is compared to a projected distance the distortion being evaluated is the average distortion along the line. For long distances greater than a few miles such comparisons are of questionable value since the distortion varies both with the distance and height difference between the endpoints.

Compatibility with the 2022 Terrestrial Reference Frame

NGS is in the process of updating the National Spatial Reference System and will replace NAD 83 with new terrestrial reference frames planned for release sometime in 2025. The TRF referenced to the North American tectonic plate will be called the *North American Terrestrial Reference Frame of 2022* (*see Smith et al., 2017*). NAVD 88 will also be replaced with a new vertical geopotential datum at the same time although it will have no effect on the terrestrial reference frames or projected coordinate systems.

In North Dakota, NATRF2022 coordinates will differ horizontally from NAD 83 by about 1.4 meters and will be lower than NAD 83 ellipsoid heights by about 0.8 meters depending on location. The GRS-80 ellipsoid will be used for NATRF2022 so the NDCRS projected coordinates will change by the same horizontal amount, and the decrease in ellipsoid height will cause a change in linear distortion by approximately 0.2 ppm. This increase is essentially negligible and will cause the magnitude of distortion to increase slightly in locations where it is positive and decrease slightly where it is negative. Thus, the NDCRS will perform equally well when referenced to NATRF2022.

The NDCRS was developed to be part of the SPCS 2022. The design criteria and parameters were developed in keeping with NGS guidelines for SPCS 2022 as previously outlined.

Concluding Remarks

Summary of NDCRS Design

The North Dakota Coordinate Reference System is a 16-zone system of low-distortion projections that optimally minimize map projection linear distortion at the topographic surface for the entire state of North Dakota. A criterion of ±20 ppm (±0.1 ft/mile) distortion was used for design of the NDCRS and was achieved for 95% of the state area.

NDCRS zones consist of 2 to 5 aggregated counties which zone boundaries correspond to county boundaries. As part of the design process a 6-mile buffer was applied around each zone to give options for projects that cross zone boundaries. Due to variations including county distribution, size and shape, distortion near zone boundaries may be greater within a zone than it is outside of an adjacent zone. To achieve the best distortion performance for projects located near zone boundaries it is recommended that the project area be examined prior to selecting the NDCRS zone. This can be done using the maps in Appendix A or by overlaying the project area on the distortion rasters in a GIS application. In some cases a project may be too long to assign to a single zone and it may be better to split the project between two zones rather than use a single zone. These instances should not be a common occurrence and is recommended they be evaluated on a case-by-case basis.

Adoption of the NDCRS by the North Dakota Department of Transportation replaces the previous method of scaling SPCS 83 coordinates to ground on the topographic surface. Since these coordinate systems were based on a modification of SPCS they were not defined using standard projection parameters which made them difficult to use consistently across geospatial platforms (e.g., in surveying, engineering, CAD, and GIS software). In contrast the NDCRS is limited to 16 projected coordinate systems which are rigorously defined using two existing projections, the Transverse Mercator and the (one-parallel) Lambert Conformal Conic which are both supported by a wide range of software platforms. Importantly they are also available for creating standardized projected coordinate system definitions in the European Petroleum Survey Group *Geodetic Parameter Registry* (2020). The EPSG registry is a freely available database of coordinate system definitions that is widely accessed by the software industry and government organizations. The registry provides machine-readable definitions in either or both Well-Known Text and Geography Markup Language formats. The NDCRS will be submitted to the EPSG registry by NGS as a part of SPCS 2022. This will aid in the adoption of the NDCRS into commercial software packages.

LDP coordinate systems have been defined and officially adopted by government agencies throughout the US. Most of these systems will also be included in SPCS 2022. Documents for several of the systems that have been adopted are listed in the references section of this report and available online: *Oregon Coordinate Reference System* (Armstrong, 2017); *Iowa Regional Coordinate System* (Dennis, 2014); *Indiana Geospatial Coordinate System* (Badger, 2016); *Wisconsin Coordinate Reference Systems* (Wisconsin State Cartographer's Office, 2012); *Rocky Mountain Tribal Coordinate Reference System* (Dennis, 2014); *Pima County Coordinate System* (Dennis, 2017); *Kansas Regional Coordinate System* (Dennis, 2017).

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Appendices

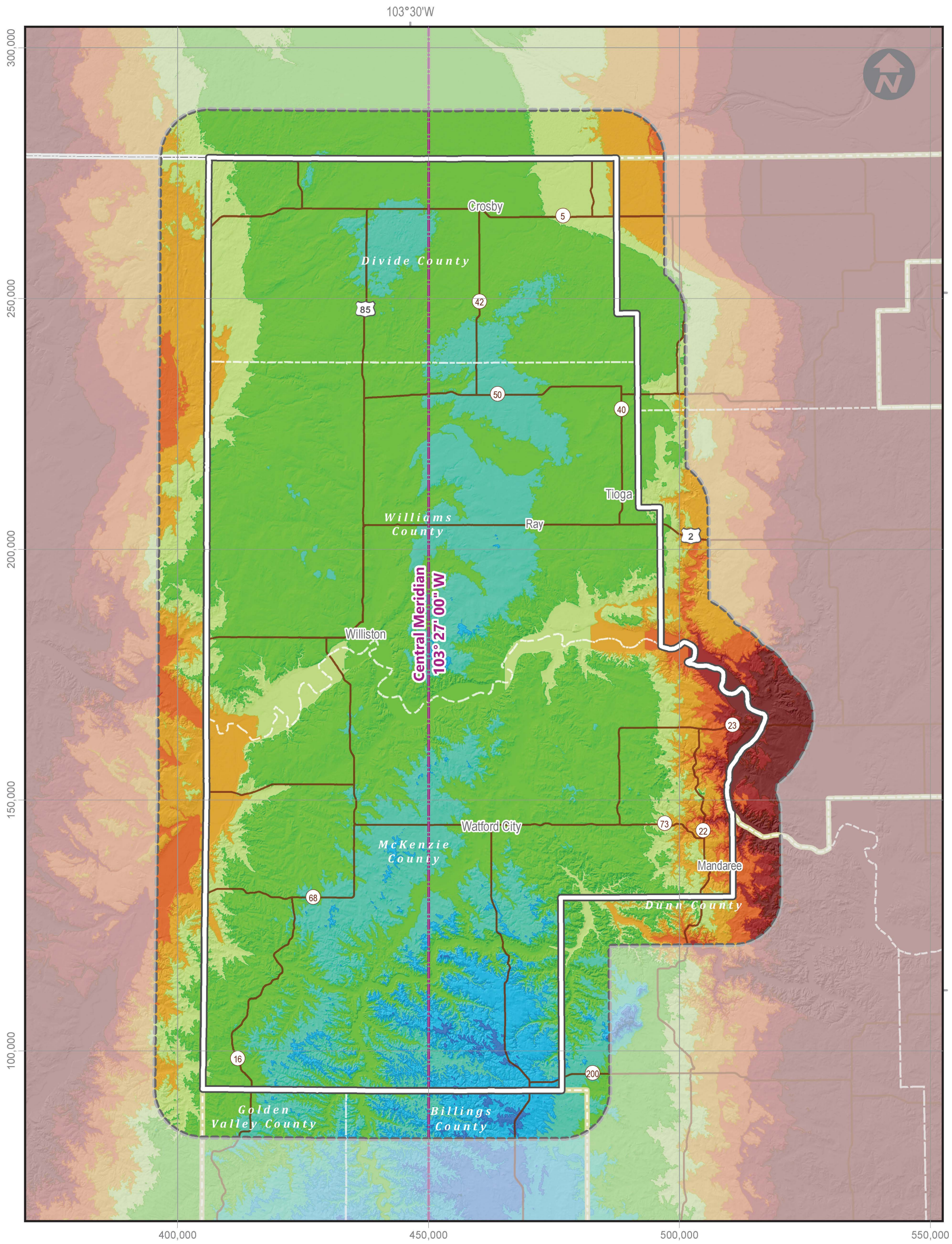
Appendix A: Distortion maps

Appendix B: Distortion and ellipsoid height values for select incorporated places

Appendix C: Equations for computing projection grid point scale factors

Appendix A. Distortion maps

Distortion maps depict the linear distortion in parts per million (ppm) in increments of 10 ppm. The maps in this report have been scaled to fit within the limitations of the page size.



North Dakota Zone 1: Williston

Transverse Mercator

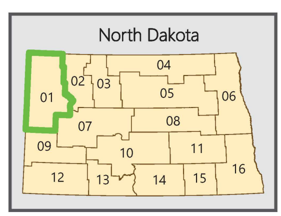
Linear Unit: ift
Latitude of Grid Origin: 46° 30' N
Central Meridian: 103° 27' W
False Northing: 0
False Easting: 1,500,000
Central Meridian Scale: 1.000092

Linear Distortion at the Topographic Surface
in parts per million (ppm)

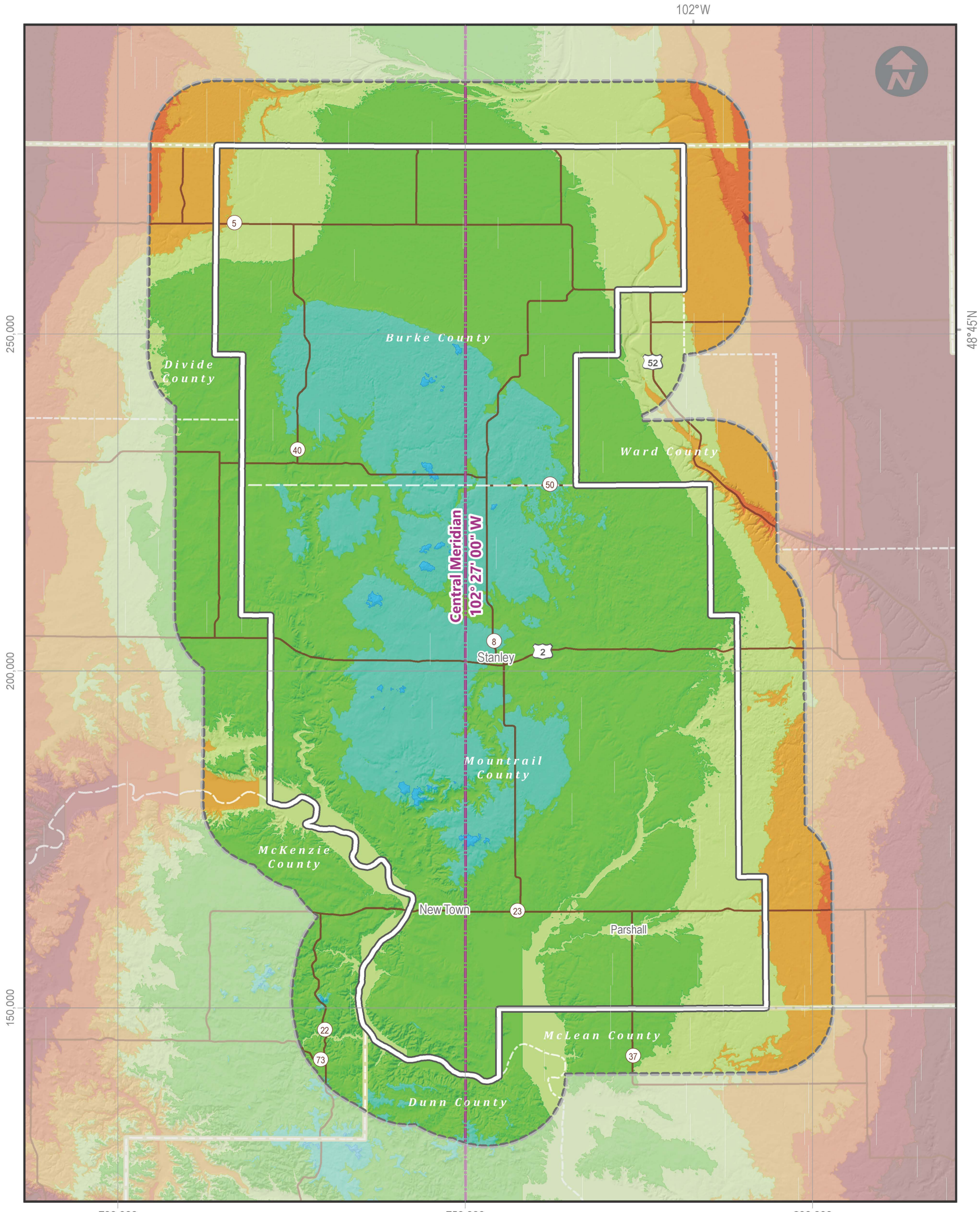
< -50 | to -50 | to -40 | to -30 | to -20 | ±10 | to +20 | to +30 | to +40 | to +50 | > + 50

LDP Zone | 6-mile Buffer | Road | Other Zone | County

Scale 1:750,000
0 5 10 Miles
0 10 20 Kilometers



NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 2: New Town

Transverse Mercator

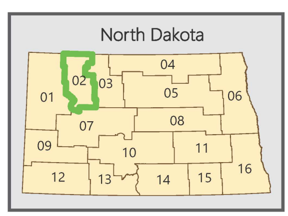
Linear Unit: ift
Latitude of Grid Origin: 46° 30' N
Central Meridian: 102° 27' W
False Northing: 0
False Easting: 2,500,000
Central Meridian Scale: 1.000094

Linear Distortion at the Topographic Surface
in parts per million (ppm)

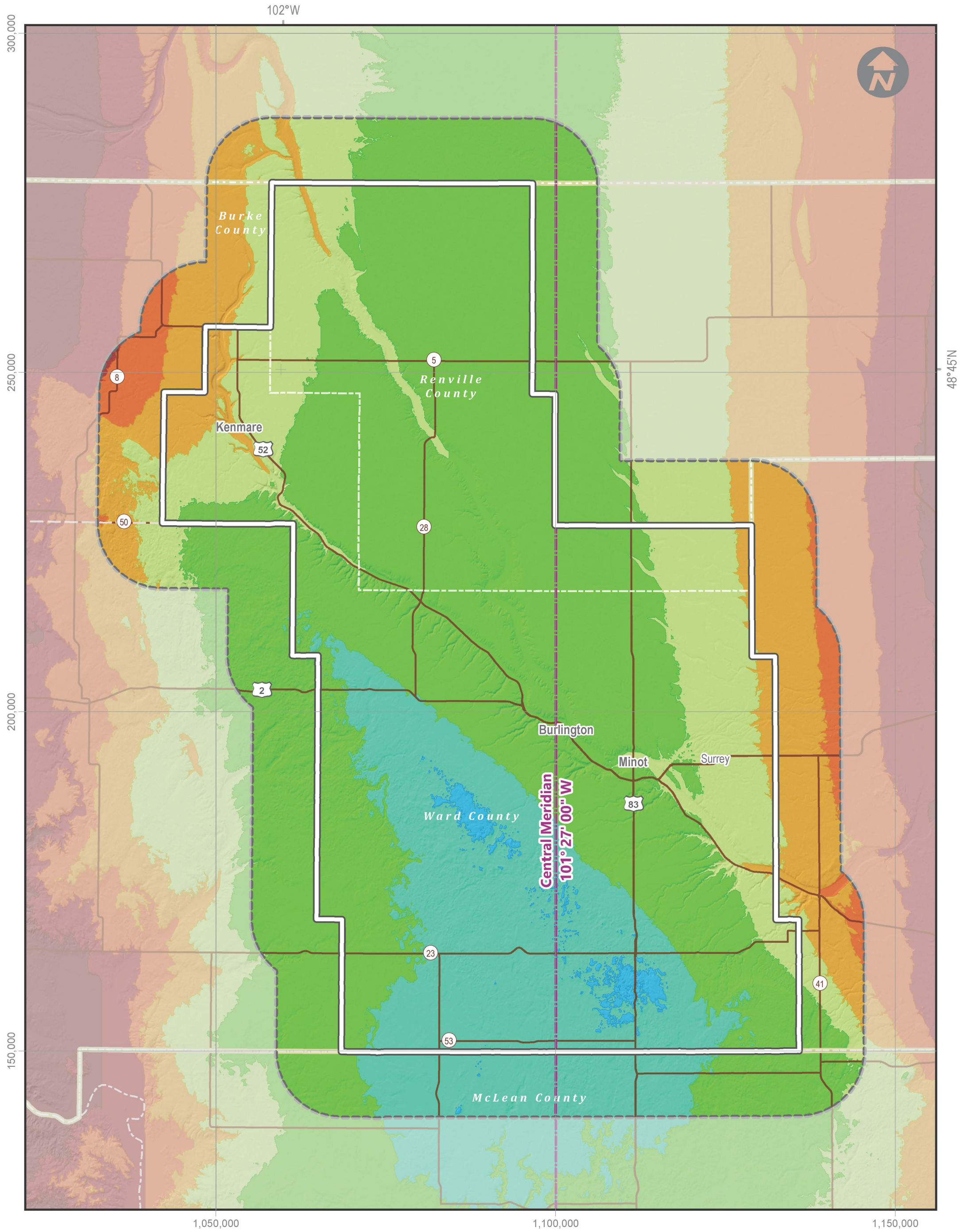
< -50 | to -30 | to +20 | to +50
to -50 | to -20 | to +30 | > + 50
to -40 | ±10 | to +40

LDP Zone | 6-mile Buffer | Road
Other Zone | County

Scale 1:550,000
0 5 10 Miles
0 10 20 Kilometers



NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 3: Minot

Transverse Mercator

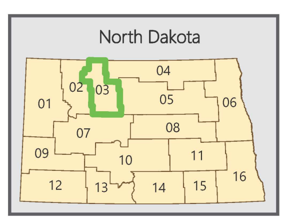
Linear Unit: ift
Latitude of Grid Origin: 46° 30' N
Central Meridian: 101° 27' W
False Northing: 0
False Easting: 3,625,000
Central Meridian Scale: 1.000080

Linear Distortion at the Topographic Surface
in parts per million (ppm)

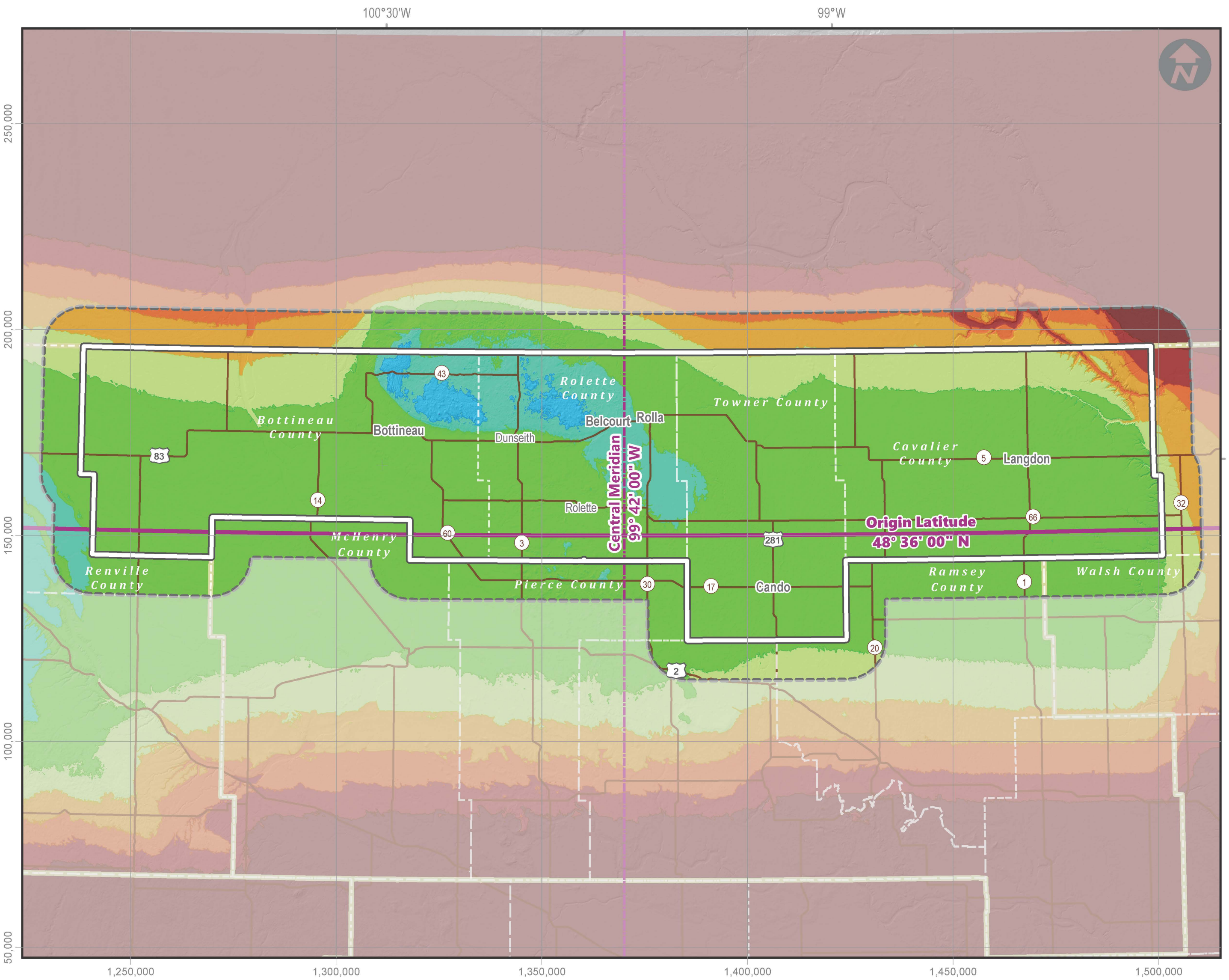
< -50	to -30	to +20	to +50
to -50	to -20	to +30	> + 50
to -40	±10	to +40	

LDP Zone — 6-mile Buffer — Road
Other Zone — County

Scale 1:550,000
0 5 10 Miles
0 10 20 Kilometers



NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 4: Bottineau

Lambert Conformal Conic

Linear Unit: ift
Latitude of Grid Origin: 48° 36' N
Central Meridian: 99° 42' W
False Northing: 500,000
False Easting: 4,500,000
Central Meridian Scale: 1.000066

Linear Distortion at the Topographic Surface
in parts per million (ppm)

< -50, to -50, to -40, to -30, to -20, ±10, to +20, to +30, to +40, to +50, > + 50

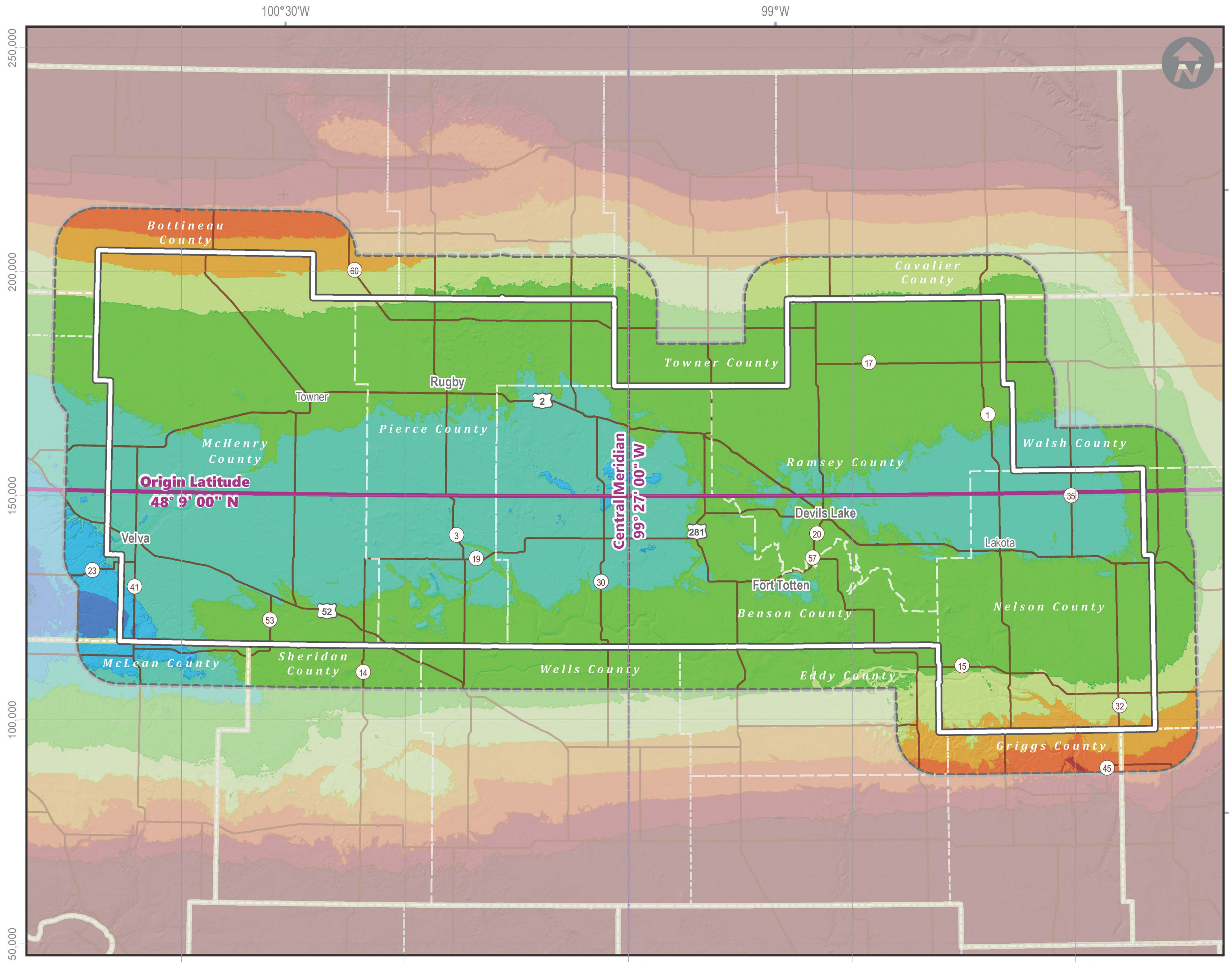
LDP Zone, 6-mile Buffer, Other Zone, County, Road

Scale 1:925,000

Miles 0 5 10 15 20
Kilometers 0 10 20 30

North Dakota

NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 5: Devils Lake

Lambert Conformal Conic

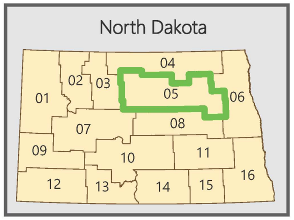
Linear Unit: ift
Latitude of Grid Origin: 48° 09' N
Central Meridian: 99° 27' W
False Northing: 500,000
False Easting: 5,500,000
Central Meridian Scale: 1.000056

Linear Distortion at the Topographic Surface
in parts per million (ppm)

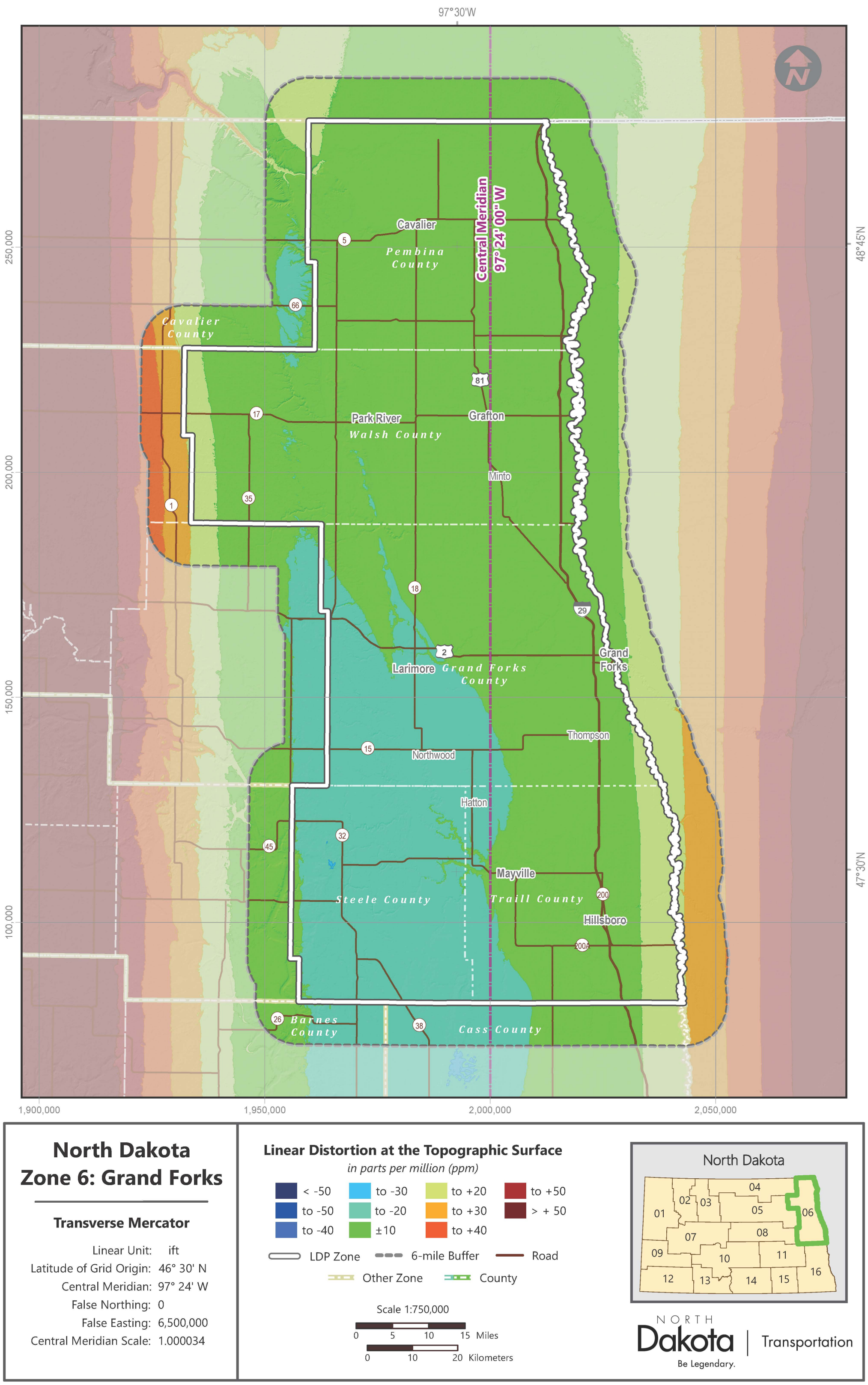
< -50	to -20	to +40
to -50	±10	to +50
to -40	to +20	> + 50
to -30	to +30	

LDP Zone
6-mile Buffer
Other Zone
County
Road

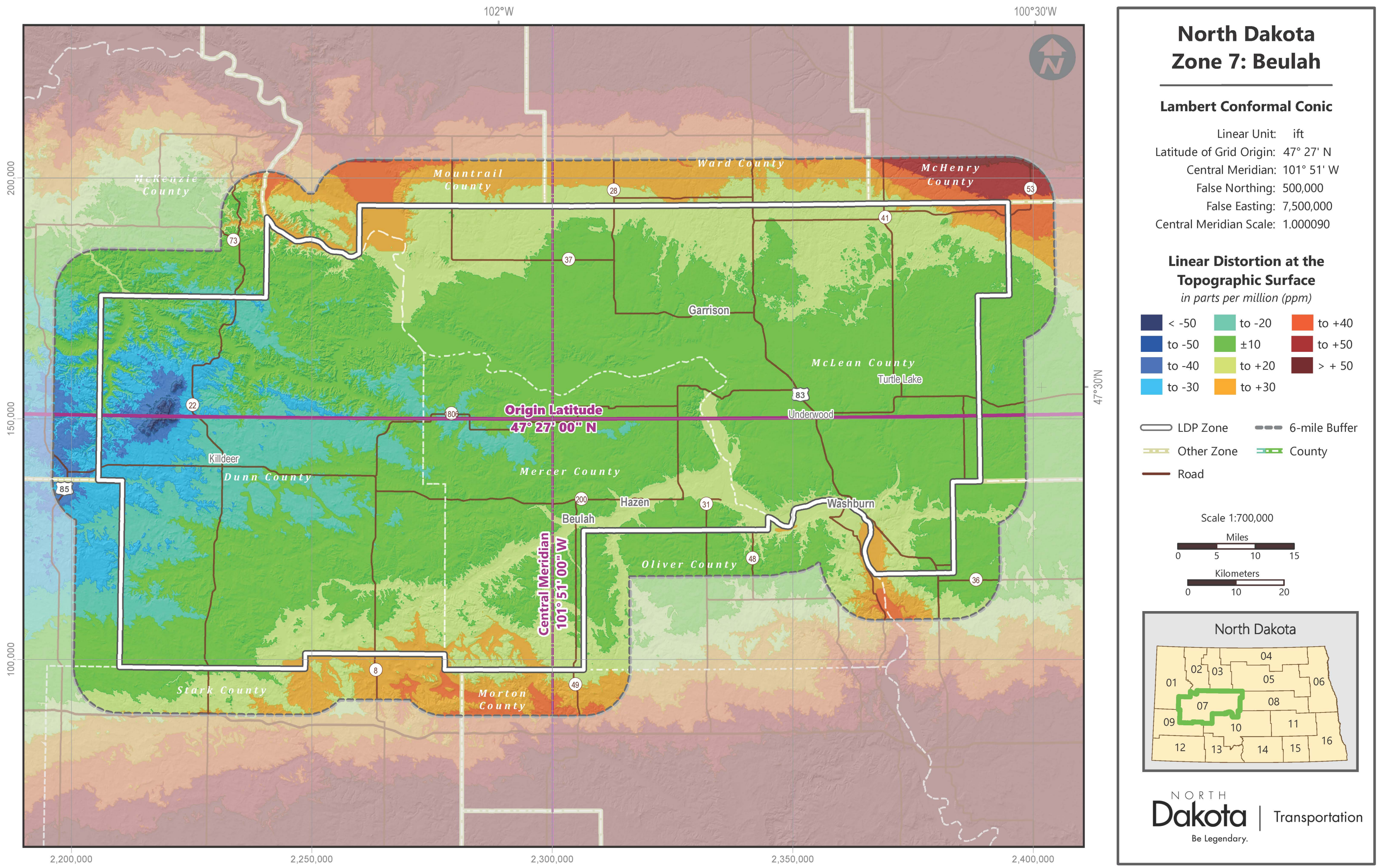
Scale 1:850,000
Miles 0 5 10 15
Kilometers 0 10 20 30

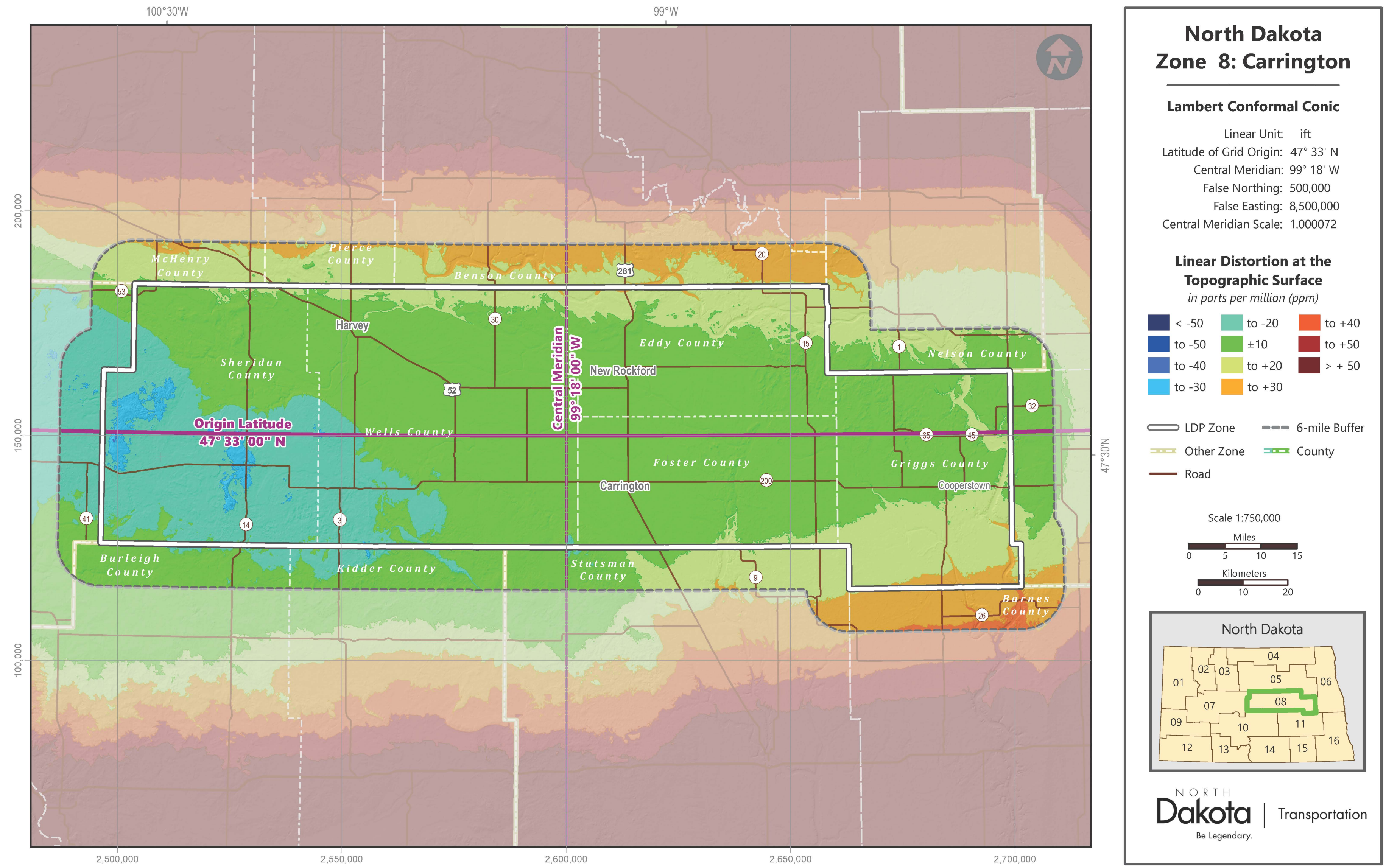


NORTH Dakota | Transportation
Be Legendary.

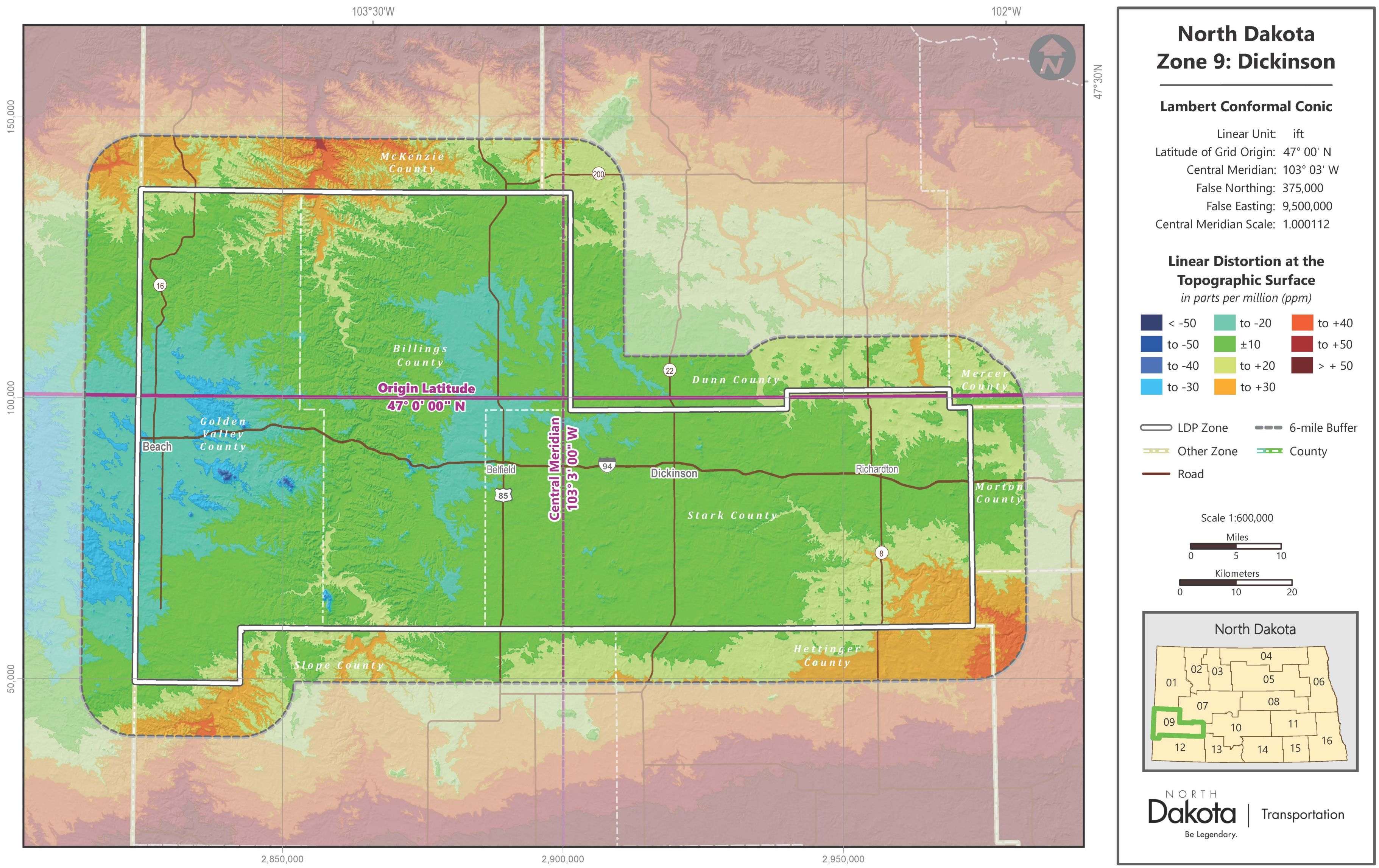


97°30'W
Central Meridian
97° 24' 00" W
48°45'N
47°30'N
250,000
200,000
150,000
100,000
1,900,000
1,950,000
2,000,000
2,050,000
Cavalier
Pembina County
Cavalier County
Park River
Grafton
Walsh County
Minto
Larimore
Grand Forks County
Grand Forks
Thompson
Northwood
Hatton
Mayville
Steele County
Traill County
Hillsboro
Barnes County
Cass County
North Dakota
Zone 6: Grand Forks
Transverse Mercator
Linear Unit: ift
Latitude of Grid Origin: 46° 30' N
Central Meridian: 97° 24' W
False Northing: 0
False Easting: 6,500,000
Central Meridian Scale: 1.000034
Linear Distortion at the Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Road
Other Zone
County
Scale 1:750,000
0 5 10 15 Miles
0 10 20 Kilometers
North Dakota
01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16
NORTH Dakota Be Legendary. | Transportation

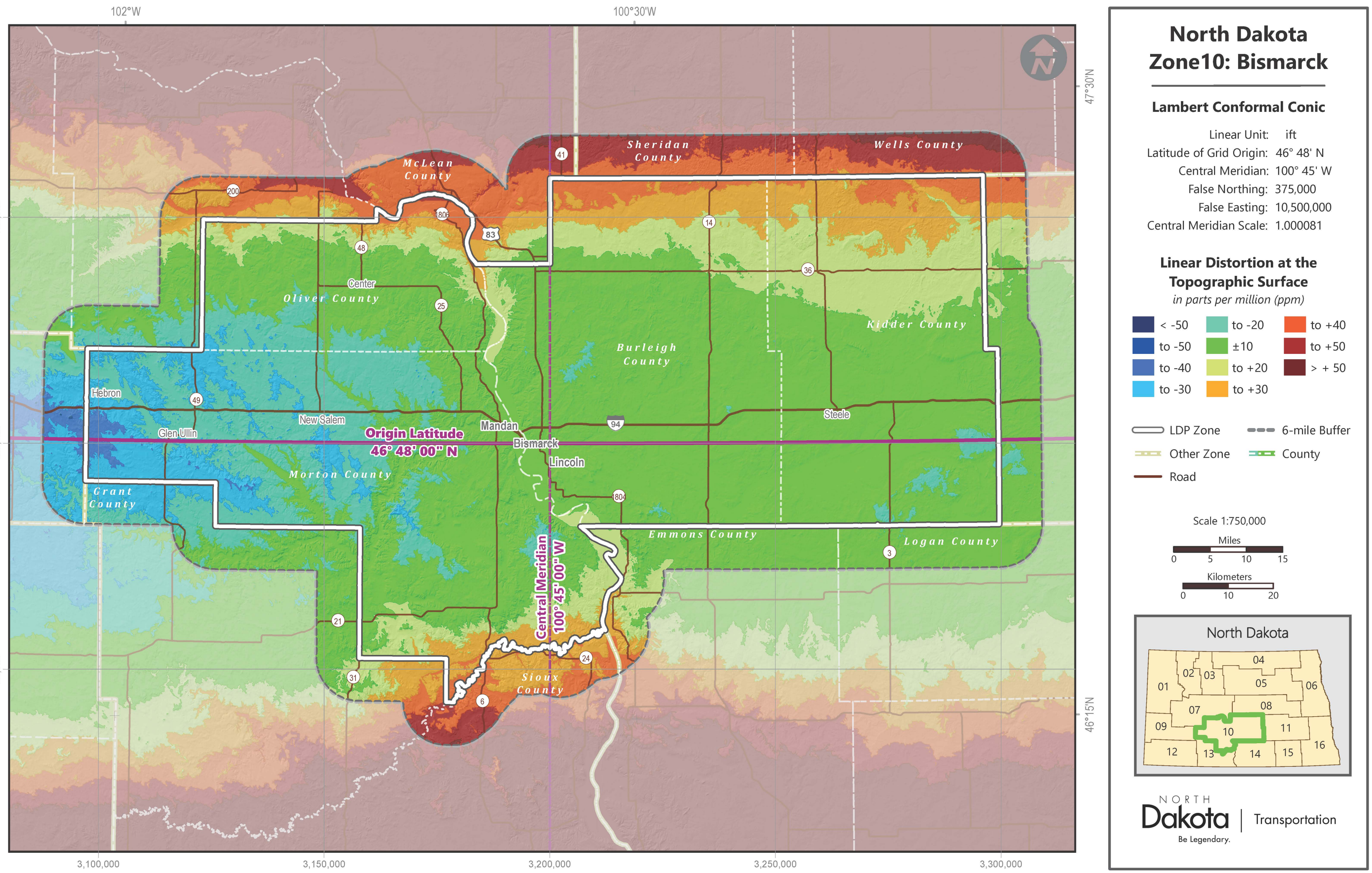

North Dakota
Zone 7: Beulah
Lambert Conformal Conic
Linear Unit: ift
Latitude of Grid Origin: 47° 27' N
Central Meridian: 101° 51' W
False Northing: 500,000
False Easting: 7,500,000
Central Meridian Scale: 1.000090
Linear Distortion at the Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Other Zone
County
Road
Scale 1:700,000
Miles
0
5
10
15
Kilometers
0
10
20
North Dakota
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
NORTH Dakota | Transportation
Be Legendary.
102°W
100°30'W
47°30'N
200,000
150,000
100,000
2,200,000
2,250,000
2,300,000
2,350,000
2,400,000
Origin Latitude
47° 27' 00" N
Central Meridian
101° 51' 00" W
McKenzie County
Mountrail County
Ward County
McHenry County
McLean County
Dunn County
Mercer County
Oliver County
Stark County
Morton County
Garrison
Turtle Lake
Underwood
Killdeer
Hazen
Beulah
Washburn
73
28
53
41
37
22
83
806
200
31
85
48
36
8
49

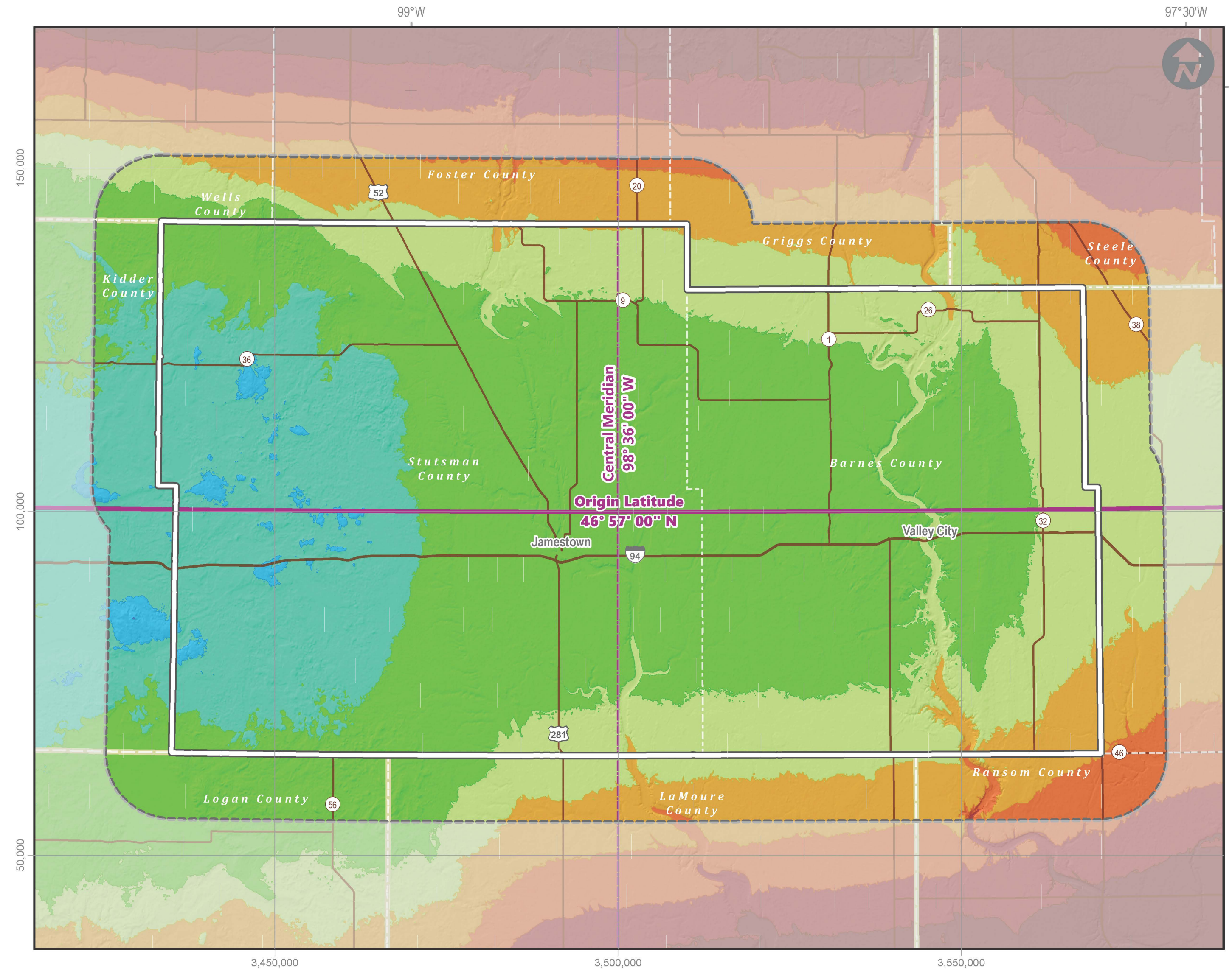


100°30'W
99°W
47°30'N
200,000
150,000
100,000
2,500,000
2,550,000
2,600,000
2,650,000
2,700,000
McHenry County
Pierce County
Benson County
Sheridan County
Eddy County
Nelson County
Wells County
Foster County
Griggs County
Burleigh County
Kidder County
Stutsman County
Barnes County
Harvey
New Rockford
Carrington
Cooperstown
Origin Latitude
47° 33' 00" N
Central Meridian
99° 18' 00" W
North Dakota
Zone 8: Carrington
Lambert Conformal Conic
Linear Unit: ift
Latitude of Grid Origin: 47° 33' N
Central Meridian: 99° 18' W
False Northing: 500,000
False Easting: 8,500,000
Central Meridian Scale: 1.000072
Linear Distortion at the
Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Other Zone
County
Road
Scale 1:750,000
Miles
0 5 10 15
Kilometers
0 10 20
North Dakota
01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16
NORTH
Dakota
Transportation
Be Legendary.



North Dakota
Zone 9: Dickinson
Lambert Conformal Conic
Linear Unit: ift
Latitude of Grid Origin: 47° 00' N
Central Meridian: 103° 03' W
False Northing: 375,000
False Easting: 9,500,000
Central Meridian Scale: 1.000112
Linear Distortion at the Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Other Zone
County
Road
Scale 1:600,000
Miles
0
5
10
Kilometers
0
10
20
North Dakota
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
NORTH Dakota | Transportation
Be Legendary.
103°30'W
102°W
47°30'N
150,000
100,000
50,000
2,850,000
2,900,000
2,950,000
McKenzie County
Billings County
Golden Valley County
Dunn County
Mercer County
Stark County
Morton County
Slope County
Hettinger County
Beach
Belfield
Dickinson
Richardton
Origin Latitude
47° 0' 00" N
Central Meridian
103° 3' 00" W


North Dakota
Zone10: Bismarck
Lambert Conformal Conic
Linear Unit: ift
Latitude of Grid Origin: 46° 48' N
Central Meridian: 100° 45' W
False Northing: 375,000
False Easting: 10,500,000
Central Meridian Scale: 1.000081
Linear Distortion at the Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Other Zone
County
Road
Scale 1:750,000
Miles
0 5 10 15
Kilometers
0 10 20
North Dakota
01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16
NORTH Dakota | Transportation
Be Legendary.
Origin Latitude
46° 48' 00" N
Central Meridian
100° 45' 00" W
Sheridan County
Wells County
McLean County
Oliver County
Kidder County
Burleigh County
Morton County
Grant County
Emmons County
Logan County
Sioux County
Hebron
Glen Ullin
New Salem
Center
Mandan
Bismarck
Lincoln
Steele
102°W
100°30'W
47°30'N
46°15'N
150,000
100,000
50,000
3,100,000
3,150,000
3,200,000
3,250,000
3,300,000



North Dakota Zone 11: Jamestown

Lambert Conformal Conic

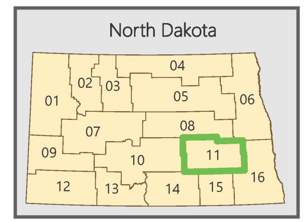
Linear Unit: ift
Latitude of Grid Origin: 46° 57' N
Central Meridian: 98° 36' W
False Northing: 375,000
False Easting: 11,500,000
Central Meridian Scale: 1.000067

Linear Distortion at the Topographic Surface
in parts per million (ppm)

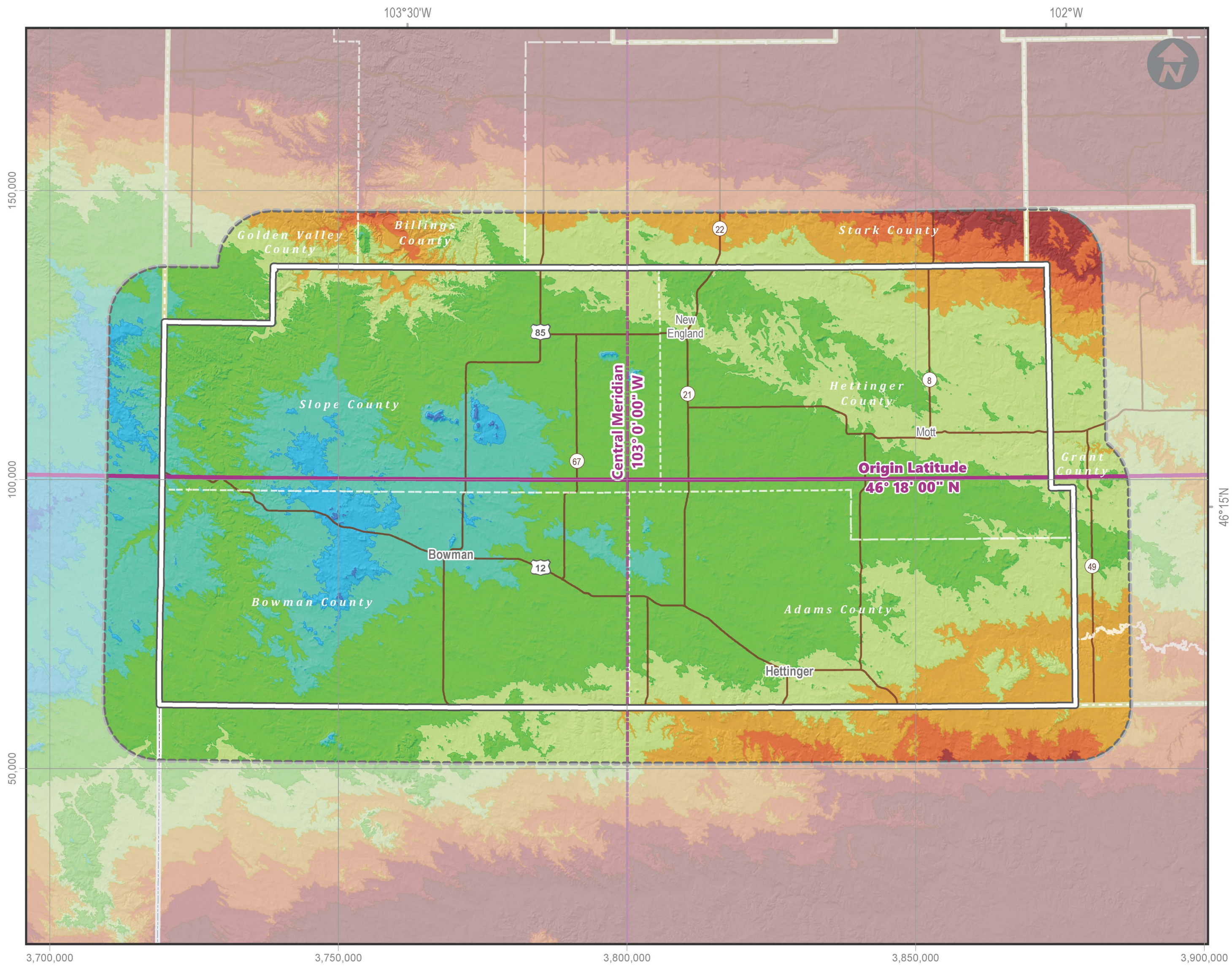
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50

LDP Zone
6-mile Buffer
Other Zone
County
Road

Scale 1:550,000
Miles 0 5 10
Kilometers 0 10 20



NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 12: Bowman

Lambert Conformal Conic

Linear Unit:	ift
Latitude of Grid Origin:	46° 18' N
Central Meridian:	103° 00' W
False Northing:	375000
False Easting:	12,500,000
Central Meridian Scale:	1.000124

Linear Distortion at the Topographic Surface

in parts per million (ppm)

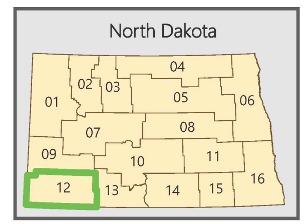
< -50	to -20	to +40
to -50	±10	to +50
to -40	to +20	> + 50
to -30	to +30	

LDP Zone
6-mile Buffer
Other Zone
County
Road

Scale 1:650,000

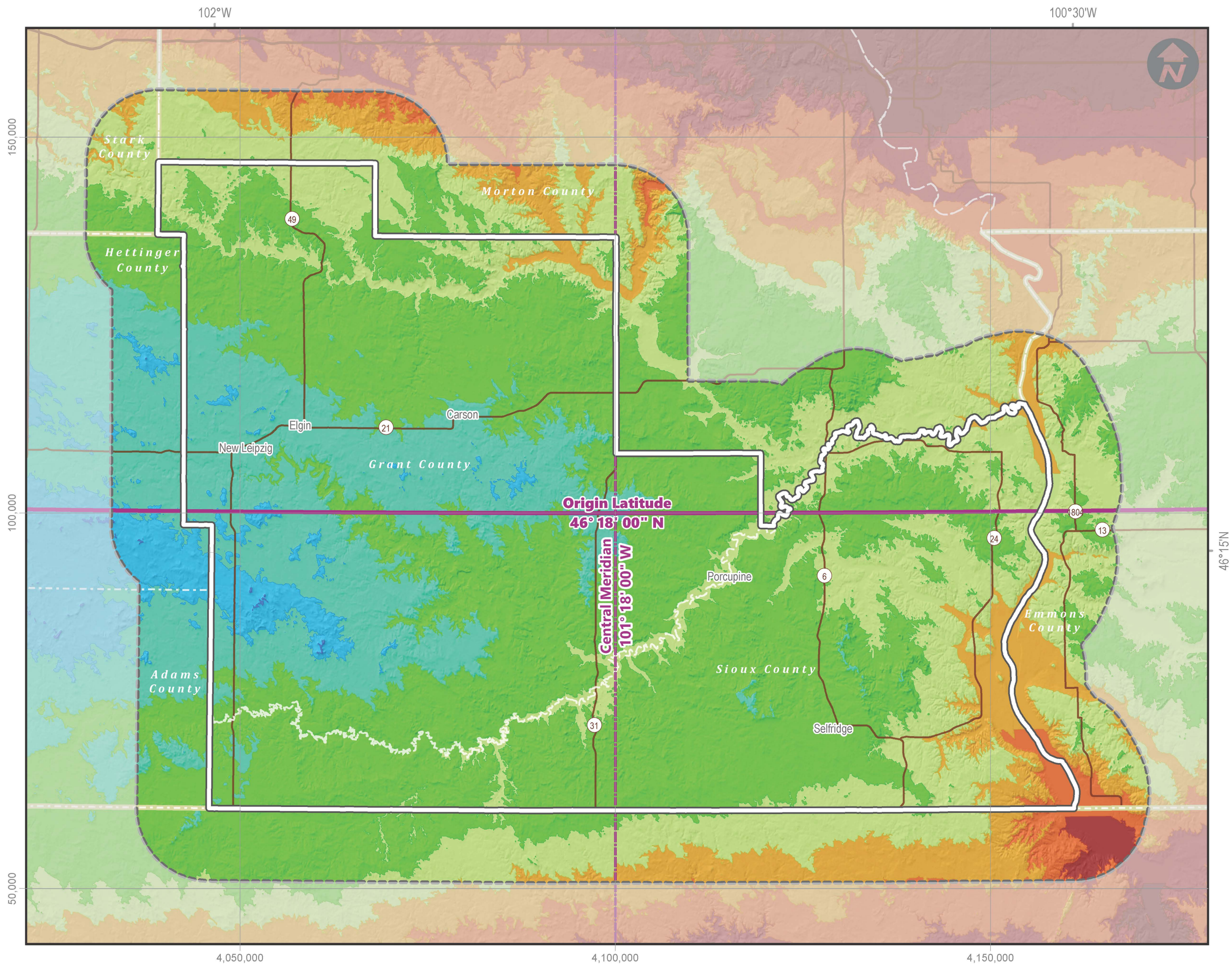
Miles 0 5 10

Kilometers 0 10 20



NORTH Dakota | Transportation

Be Legendary.



North Dakota Zone 13: Cannon Ball

Lambert Conformal Conic

Linear Unit:	ift
Latitude of Grid Origin:	46° 18' N
Central Meridian:	101° 18' W
False Northing:	375,000
False Easting:	413,500,000
Central Meridian Scale:	1.000093

Linear Distortion at the Topographic Surface

in parts per million (ppm)

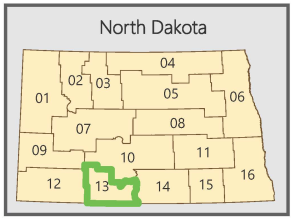
< -50	to -20	to +40
to -50	±10	to +50
to -40	to +20	> + 50
to -30	to +30	

LDP Zone
6-mile Buffer
Other Zone
County
Road

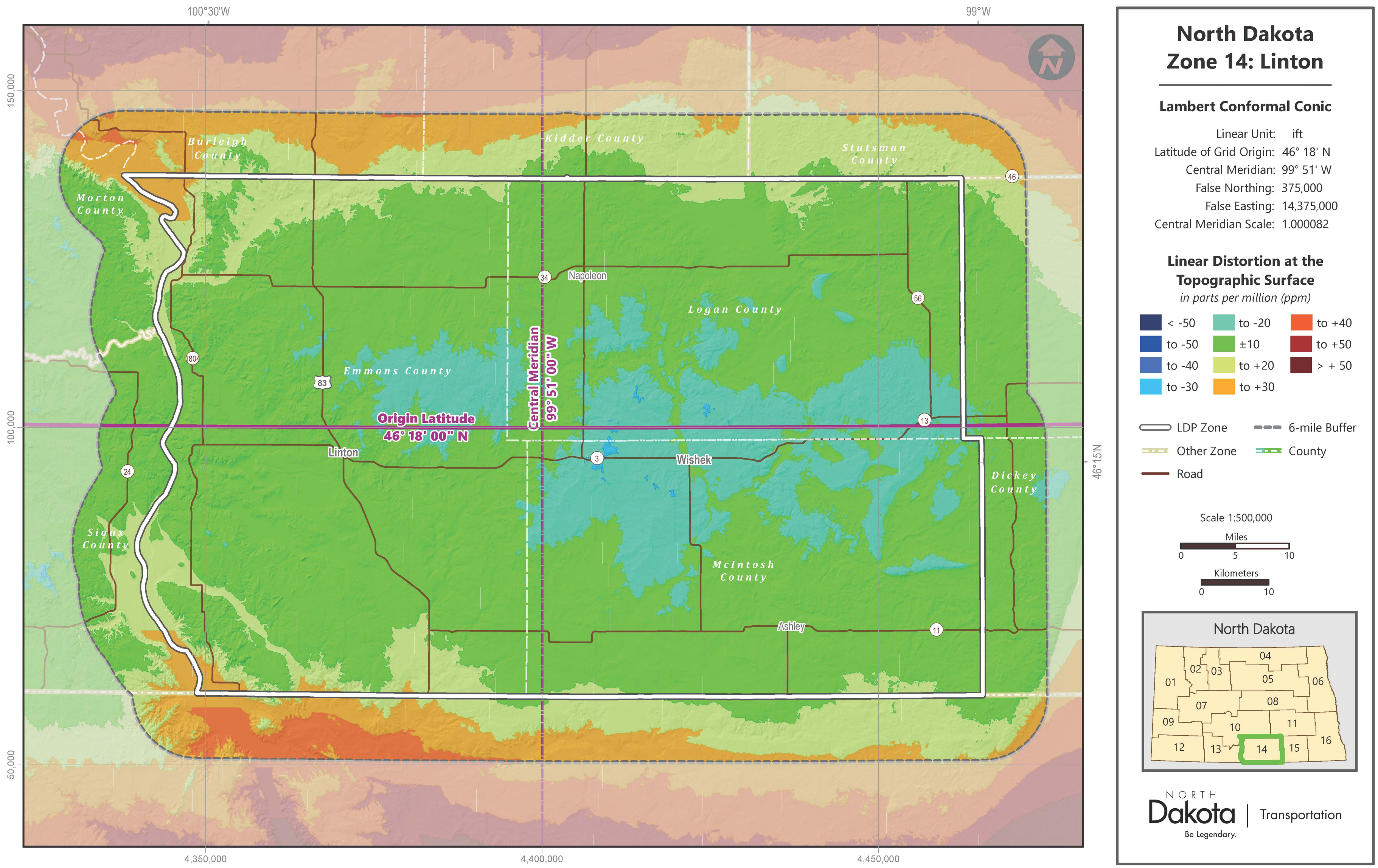
Scale 1:500,000

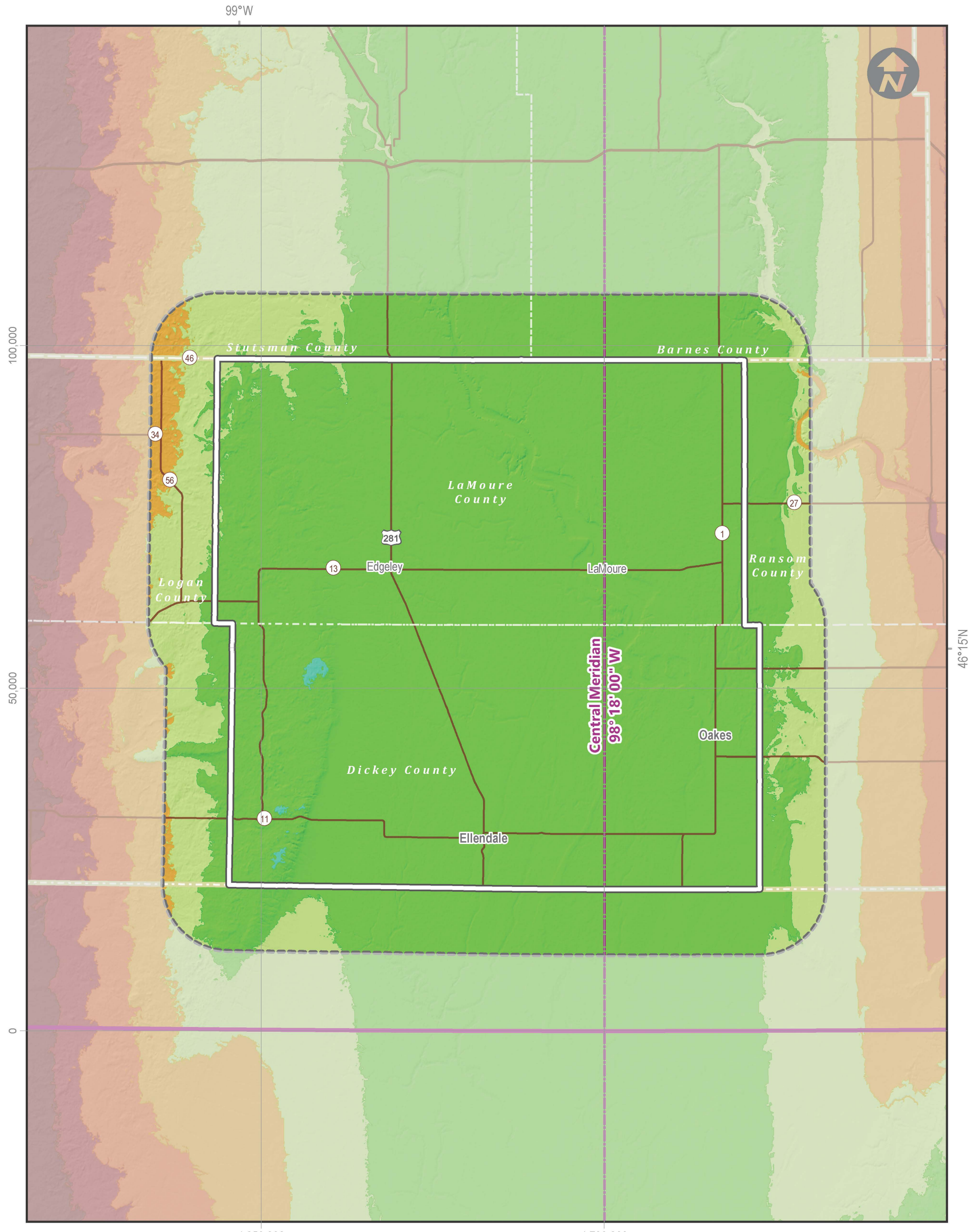
Miles 0 5 10

Kilometers 0 10



NORTH Dakota | Transportation
Be Legendary.


100°30'W
99°W
150,000
100,000
50,000
46°15'N
4,350,000
4,400,000
4,450,000
Burleigh County
Kidder County
Stutsman County
Morton County
Logan County
Emmons County
Dickey County
Sioux County
McIntosh County
Napoleon
Linton
Wishek
Ashley
Origin Latitude 46° 18' 00" N
Central Meridian 99° 51' 00" W
North Dakota
Zone 14: Linton
Lambert Conformal Conic
Linear Unit: ift
Latitude of Grid Origin: 46° 18' N
Central Meridian: 99° 51' W
False Northing: 375,000
False Easting: 14,375,000
Central Meridian Scale: 1.000082
Linear Distortion at the Topographic Surface
in parts per million (ppm)
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50
LDP Zone
6-mile Buffer
Other Zone
County
Road
Scale 1:500,000
Miles
0
5
10
Kilometers
0
10
North Dakota
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 15: Oakes

Transverse Mercator

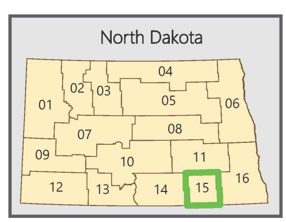
Linear Unit: ift
Latitude of Grid Origin: 45° 45' N
Central Meridian: 98° 18'W
False Northing: 0
False Easting: 15,375,000
Central Meridian Scale: 1.000061

Linear Distortion at the Topographic Surface
in parts per million (ppm)

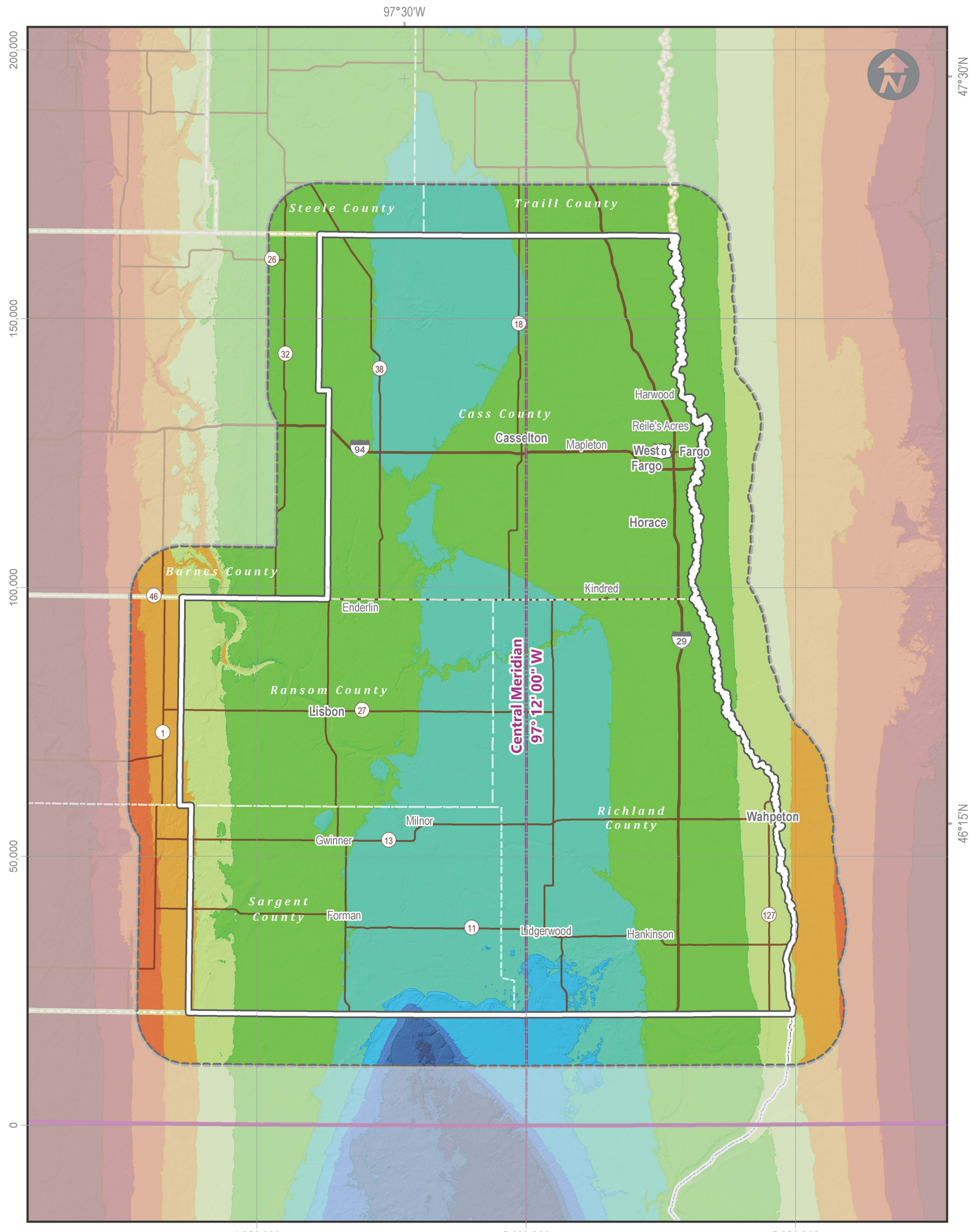
< -50
to -50
to -40
to -30
to -20
±10
to +20
to +30
to +40
to +50
> + 50

LDP Zone
6-mile Buffer
Road
Other Zone
County

Scale 1:550,000
0 5 10 Miles
0 10 20 Kilometers



NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 16: Fargo

Transverse Mercator

Linear Unit:	ift
Latitude of Grid Origin:	45° 45' N
Central Meridian:	97° 12'W
False Northing:	0
False Easting:	16,375,000
Central Meridian Scale:	1.000032

Linear Distortion at the Topographic Surface

in parts per million (ppm)

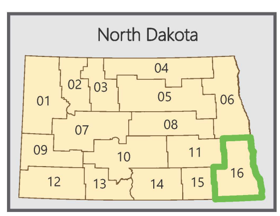
< -50	to -30	to +20	to +50
to -50	to -20	to +30	> + 50
to -40	±10	to +40	

LDP Zone
6-mile Buffer
Road
Other Zone
County

Scale 1:700,000

0 5 10 15 Miles

0 10 20 Kilometers



NORTH Dakota | Transportation

Be Legendary.

Appendix B. Distortion and ellipsoid height values for incorporated places

Table B1. Distortion and ellipsoid height values for select incorporated places

There are 357 incorporated places in North Dakota. This table represents the 94 places that have a 2019 estimated population of 500 or more. The locations represent the approximate centroid based on the 2018 boundaries according to the U.S. Census Bureau. Ellipsoid heights have been rounded to the nearest meter and foot.

Population data source: U.S. Census Bureau population 2019 estimates

Name	2019 Population Estimate	Latitude	Longitude	Ellipsoid height (m)	Ellipsoid height (ift)	Zone	Zone Name	Linear distortion (ppm)
Ashley	661	46.0341	-99.3715	596	1,954	14	Linton	-0.6539
Beach	1,043	46.9181	-104.0044	834	2,735	9	Dickinson	-17.6345
Belfield	1,012	46.8853	-103.1996	787	2,583	9	Dickinson	-9.4231
Beulah	3,139	47.2633	-101.7779	571	1,873	7	Beulah	5.7916
Bismarck	73,529	46.8083	-100.7837	499	1,636	10	Bismarck	2.8449
Bottineau	2,154	48.8272	-100.4457	491	1,611	4	Bottineau	-3.1152
Bowman	1,599	46.1831	-103.3949	885	2,903	12	Bowman	-12.6377
Burlington	1,201	48.2753	-101.4288	508	1,668	3	Minot	0.3511
Cando	1,057	48.4867	-99.2099	426	1,398	4	Bottineau	1.1636
Carrington	1,980	47.4497	-99.1262	455	1,494	8	Carrington	2.1607
Casselton	2,475	46.9005	-97.2112	258	845	16	Fargo	-8.3680
Cavalier	1,238	48.7939	-97.6223	242	795	6	Grand Forks	-0.6941
Center	578	47.1164	-101.2996	615	2,017	10	Bismarck	-0.1270
Cooperstown	903	47.4444	-98.1240	411	1,348	8	Carrington	9.2698
Crosby	1,285	48.9142	-103.2949	581	1,906	1	Williston	2.5355
Devils Lake	7,320	48.1145	-98.8585	420	1,377	5	Devils Lake	-9.5945
Dickinson	23,133	46.8792	-102.7896	741	2,431	9	Dickinson	-1.9421
Drayton	747	48.5711	-97.1778	216	707	6	Grand Forks	3.5226
Dunseith	764	48.8131	-100.0610	515	1,689	4	Bottineau	-7.7563
Edgeley	556	46.3591	-98.7157	454	1,491	15	Oakes	2.3521
Elgin	603	46.4039	-101.8460	686	2,251	13	Cannon Ball	-12.9057
llendale	1,246	46.0028	-98.5270	416	1,365	15	Oakes	-0.4229
Enderlin	838	46.6230	-97.6015	316	1,036	16	Fargo	-5.8955
Fargo	124,662	46.8772	-96.7898	246	807	16	Fargo	5.4582
Forman	504	46.1077	-97.6365	357	1,172	16	Fargo	-9.9895
Garrison	1,462	47.6522	-101.4157	565	1,853	7	Beulah	7.6965
Glen Ullin	717	46.8150	-101.8299	634	2,080	10	Bismarck	-18.3606
Grafton	4,157	48.4122	-97.4106	225	738	6	Grand Forks	-1.2230
Grand Forks	55,839	47.9253	-97.0329	225	738	6	Grand Forks	8.0118
Gwinner	937	46.2258	-97.6626	363	1,190	16	Fargo	-9.1947
Hankinson	885	46.0697	-96.9017	297	974	16	Fargo	-8.0143
Harvey	1,646	47.7697	-99.9354	465	1,527	8	Carrington	6.3909
Harwood	829	46.9794	-96.8806	243	799	16	Fargo	1.0922

Name	2019 Population Estimate	Latitude	Longitude	Ellipsoid height (m)	Ellipsoid height (ift)	Zone	Zone Name	Linear distortion (ppm)
Hatton	743	47.6397	-97.4534	298	978	6	Grand Forks	-12.5297
Hazen	2,311	47.2944	-101.6227	546	1,790	7	Beulah	8.1344
Hebron	675	46.9006	-102.0454	663	2,176	10	Bismarck	-21.4186
Hettinger	1,155	46.0014	-102.6368	813	2,667	12	Bowman	10.0973
Hillsboro	1,624	47.4039	-97.0620	249	817	6	Grand Forks	2.9627
Horace	2,944	46.7589	-96.9037	250	820	16	Fargo	-0.8840
Jamestown	15,084	46.9105	-98.7084	426	1,398	11	Jamestown	0.4450
Kenmare	1,021	48.6747	-102.0827	570	1,871	3	Minot	17.3064
Killdeer	1,144	47.3720	-102.7541	681	2,236	7	Beulah	-15.8912
Kindred	781	46.6486	-97.0170	260	852	16	Fargo	-6.2978
Lakota	625	48.0428	-98.3362	435	1,427	5	Devils Lake	-10.4225
LaMoure	883	46.3572	-98.2945	394	1,293	15	Oakes	-0.7603
Langdon	1,752	48.7600	-98.3682	468	1,535	4	Bottineau	-3.4183
Larimore	1,288	47.9067	-97.6268	314	1,030	6	Grand Forks	-11.6515
Lidgerwood	616	46.0755	-97.1518	316	1,036	16	Fargo	-17.3304
Lincoln	3,817	46.7625	-100.7004	505	1,658	10	Bismarck	2.0160
Linton	978	46.2667	-100.2329	539	1,769	14	Linton	-2.3464
Lisbon	2,051	46.4416	-97.6812	335	1,100	16	Fargo	-3.7591
Mandan	22,752	46.8267	-100.8896	511	1,678	10	Bismarck	0.9479
Mapleton	1,238	46.8891	-97.0526	249	816	16	Fargo	-5.4383
Mayville	1,802	47.4980	-97.3245	270	886	6	Grand Forks	-7.9303
Milnor	634	46.2591	-97.4562	314	1,032	16	Fargo	-12.4968
Minot	47,382	48.2325	-101.2963	486	1,595	3	Minot	5.4227
Minto	592	48.2917	-97.3715	223	731	6	Grand Forks	-0.8868
Mohall	723	48.7634	-101.5132	476	1,561	3	Minot	5.7182
Mott	724	46.3725	-102.3271	734	2,409	12	Bowman	9.6964
Napoleon	752	46.5034	-99.7620	589	1,931	14	Linton	-3.9696
New England	600	46.5392	-102.8682	794	2,605	12	Bowman	8.2090
New Rockford	1,339	47.6800	-99.1379	445	1,459	8	Carrington	4.8615
New Salem	989	46.8450	-101.4113	632	2,073	10	Bismarck	-17.7196
New Town	2,592	47.9808	-102.4902	582	1,909	2	New Town	2.9263
Northwood	895	47.7342	-97.5668	311	1,020	6	Grand Forks	-12.7942
Oakes	1,701	46.1386	-98.0904	375	1,230	15	Oakes	5.4357
Park River	1,338	48.3986	-97.7412	283	929	6	Grand Forks	-2.5204
Parshall	1,288	47.9533	-102.1349	605	1,985	2	New Town	6.0153
Pembina	544	48.9664	-97.2437	213	697	6	Grand Forks	2.2977
Portland	595	47.4983	-97.3704	276	906	6	Grand Forks	-9.2114
Ray	901	48.3445	-103.1652	686	2,251	1	Williston	-10.0376
Reile's Acres	700	46.9269	-96.8665	245	804	16	Fargo	1.5295
Richardton	561	46.8839	-102.3157	705	2,312	9	Dickinson	3.5645
Rolette	593	48.6608	-99.8415	472	1,549	4	Bottineau	-7.4430
Rolla	1,252	48.8578	-99.6179	536	1,759	4	Bottineau	-7.9114
Rugby	2,590	48.3689	-99.9962	450	1,477	5	Devils Lake	-7.2658

Name	2019 Population Estimate	Latitude	Longitude	Ellipsoid height (m)	Ellipsoid height (ift)	Zone	Zone Name	Linear distortion (ppm)
Stanley	2,677	48.3172	-102.3905	668	2,192	2	New Town	-10.4639
Steele	711	46.8547	-99.9159	532	1,745	10	Bismarck	-1.9277
Surrey	1,393	48.2364	-101.1335	469	1,539	3	Minot	13.2873
Thompson	1,021	47.7736	-97.1098	237	778	6	Grand Forks	2.6203
Tioga	1,339	48.3972	-102.9382	683	2,241	1	Williston	2.5962
Towner	526	48.3458	-100.4054	428	1,403	5	Devils Lake	-5.1860
Turtle Lake	554	47.5200	-100.8901	551	1,807	7	Beulah	4.3937
Underwood	735	47.4564	-101.1371	588	1,929	7	Beulah	-2.1475
Valley City	6,323	46.9233	-98.0032	386	1,265	11	Jamestown	6.6612
Velva	1,190	48.0561	-100.9293	468	1,534	5	Devils Lake	-15.9423
Wahpeton	7,734	46.2652	-96.6059	265	870	16	Fargo	16.2237
Walhalla	907	48.9233	-97.9181	294	963	6	Grand Forks	5.7092
Washburn	1,264	47.2892	-101.0290	531	1,742	7	Beulah	10.6750
Watford City	7,835	47.8022	-103.2832	630	2,067	1	Williston	-4.8258
West Fargo	37,058	46.8750	-96.9004	246	808	16	Fargo	-0.2162
Williston	29,033	48.1470	-103.6180	560	1,837	1	Williston	6.1504
Wilton	716	47.1586	-100.7835	607	1,991	7	Beulah	7.7508
Wishek	891	46.2569	-99.5571	610	2,002	14	Linton	-13.3698

Appendix C. Equations for computing projection grid point scale factors

The projection grid point scale factor, k, is required to compute map projection distortion for a point on the ground. Because some surveying, engineering, and GIS software does not provide k, formulas for computing it are given below for the Transverse Mercator and Lambert Conformal Conic projections. These equations were derived from those provided in *NOAA Manual NOS NGS 5* "State Plane Coordinate System of 1983" by James Stem (1990). Equations for computing the convergence angle of these projections are also provided.

For the Transverse Mercator projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 32-35):

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1 - e^2} \right) \left[1 + \frac{(\Delta\lambda \cos\varphi)^2}{12} \left(5 - 4\tan^2\varphi + \frac{e^2\cos^2\varphi}{1-e^2} \left(9 - 24\tan^2\varphi \right) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians, for negative west longitude)

λ_0 = central meridian longitude

λ = geodetic longitude of point

φ = geodetic latitude of point

e^2 = reference ellipsoid first eccentricity squared

The following shorter equation can be used to approximate k for the Transverse Mercator projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about ±1° of the central meridian (about 50 to 60 miles between latitudes of 30° and 45°):

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1 - e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing k throughout a UTM system zone (at the zone width of ±3° from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about ±1° of the central meridian):

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0\,R)^2}$$

where E = Easting of the point where k is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R = Earth geometric mean radius of curvature

For the Lambert Conformal Conic projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos\varphi_C}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_C}}\,\exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C} - \ln\frac{1+\sin\varphi}{1-\sin\varphi} + e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi} - \ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right)\right]\right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if k_0 = 1)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. To use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_C and k_0. The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_C = \sin^{-1}\left[\frac{2\ln\left(\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}\right)}{\ln\left(\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}\right) + e\left[\ln\left(\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}\right) - \ln\left(\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]}\right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_C}\sqrt{\frac{1-e^2\sin^2\varphi_0}{1-e^2\sin^2\varphi_N}}$$

$$\times\ \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\left(\frac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\frac{1+\sin\varphi_C}{1-\sin\varphi_C}\right) + e\left(\ln\left[\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right] - \ln\left[\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right]\right)\right]\right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda\,\sin\varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than ±00.2″ if the computation point is within ~1° of the central meridian. For any LCC, the convergence angle is exactly $\gamma = -\Delta\lambda\,\sin\varphi_C$.